DELAY AND RANDOM SCATTERING ESTIMATION WITH A BAND-LIMITED SIGNAL: UNCONDITIONAL CRB AND MLE

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ABSTRACT

Time-delay and source power estimation are fundamental tasks in a plethora of applications, in particular for remote sensing systems where the goal is to characterize the reflecting surface. Standard processing considers a Gaussian conditional signal model for which all the unknown parameters are assumed to be deterministic, but it may be more informative to consider a Gaussian random surface scattering, leading to an unconditional signal model. In this contribution, compact closed-form unconditional Cramér-Rao bound (CRB) expressions for delay and Gaussian source variance estimation are provided, considering a generic band-limited signal, as well as the corresponding maximum likelihood estimators (MLEs). The CRBs are expressed in terms of the signal samples, making it especially easy to use whatever baseband signal is considered. The results are validated with two representative band-limited signals to support the discussion.

***Index Terms*—** Unconditional CRB, unconditional MLE, delay estimation, random scattering, band-limited signals.

1. INTRODUCTION

Parameter estimation is a problem of interest in several Earth observation applications, e.g., radar, Global Navigation Satellite Systems (GNSS), or GNSS-based reflectometry (GNSS-R) [1–4]. The latter is a powerful way to retrieve information from a reflecting surface by exploiting GNSS as signals of opportunity, to obtain either altimetric information or the reflecting surface characterization (e.g., roughness, surface wind, soil moisture, sea-ice salinity or snow water content) [3, 4].

Standard GNSS-R processing, as typically considered also in GNSS-based positioning, assumes a Gaussian conditional signal model (CSM) [5] for both direct and reflected paths [6]. In such case, signal delay, Doppler, complex amplitude and noise power, for both paths, are assumed to be unknown deterministic parameters that must be estimated. The optimal estimation performance, in the mean square error (MSE) sense, is given by the conditional Cramér-Rao bound (CRB_c) [7, 8], for which useful closed-form expressions are given in [9–11]. The corresponding estimator is the conditional maximum likelihood (ML) estimator (CMLE), which is known to be asymptotically efficient (i.e., in the large sample and/or high signal-to-noise (SNR) regimes [5, 12]). Notice that GNSS-R processing is first applied in a coherent manner for a low number of GNSS codes (i.e., limited by data bit transitions) and then using non-coherent integration strategies [3, 13] to obtain the delay/Doppler maps (i.e., because of the extremely low power received from the reflecting surface), which are further processed in subsequent stages.

In this contribution, in contrast to the standard deterministic CSM case, an unconditional signal model (USM) [5] is proposed in order to account for a possibly random surface scattering. This reformulation of the GNSS-R problem may allow to extract a more accurate information of the reflecting surface power to build surface maps (as done in airborne or spaceborne synthetic-aperture radar (SAR) [14, 15]), or to build change detector metrics [15], which could be exploited to charcterize the transition between different surfaces.

The goal is then twofold: i) derive closed-form unconditional CRB (CRB_u) expressions (in the vein of [9–11]) for delay and Gaussian source variance estimation, and ii) compare these expressions to the corresponding unconditional ML estimator (UMLE) [16–19]. A generic band-limited signal is considered, then obtaining general expressions of interest in different applications. Notice that the CRBs are expressed in terms of the signal samples, making it especially easy to use whatever the baseband signal considered. The results are validated with two representative band-limited signals (i.e., GNSS and radar) to support the discussion.

2. SIGNAL MODEL

Let us consider the transmission of a band-limited signal $s(t)$ with bandwidth B, from a transmitter T at position $\mathbf{p}_T(t)$ to a receiver R at position $\mathbf{p}_R(t)$ (through a reflection), over a carrier with frequency f_c (wavelength $\lambda_c = \frac{c}{f_c}$, c the speed of light). If the T-to-R distance is constant during the observation time (constant propagation delay), $\|\mathbf{p}_R(t) - \mathbf{p}_T(t - \tau(t))\| = c\tau$, the baseband output of the

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receiver front-end is,

$$y(t) = \alpha s(t-\tau) + n(t), \quad (1)$$

where $n(t)$ is a complex white Gaussian noise with variance σ_n^2 and α is a complex amplitude. Notice that α depends mainly on i) the reflecting surface, and ii) the transmitted signal power, the transmitter/receiver antenna gains and polarization vectors, and the propagation path length between T and R (including reflection) [3, 14, 15]. The discrete signal model is built from K snapshots of N samples at $T_s = 1/F_s$, with $F_s \geq B$ the Hilbert filter bandwidth,

$$\mathbf{y}_k = \alpha_k \mathbf{s}(\tau) + \mathbf{n}_k, \; k = 1, \ldots, K, \quad (2)$$

with $\mathbf{y}_k^T = (y_k(T_s), \ldots, y_k(NT_s))$ the k-th snapshot received signal samples, $\mathbf{n}_k^T = (n_k(T_s), \ldots, n_k(NT_s))$, the noise samples and $\mathbf{s}^T(\tau) = (s(T_s - \tau), \ldots, s(NT_s - \tau))$, the delayed transmitted signal samples. As mentioned before, the noise is assumed Gaussian $\mathbf{n}_k \sim \mathcal{CN}(\mathbf{0}, \sigma_n^2 \mathbf{I}_N)$. Depending on the definition of α, two cases are of interest in this study:

• CSM: the standard case, where α_k (i.e., snapshot k) is assumed to be an unknown complex parameter. Then,

$$\mathbf{y}_k \sim \mathcal{CN}(\alpha_k \mathbf{s}(\tau), \sigma_n^2 \mathbf{I}_N), \quad (3)$$

and the unknown deterministic parameters are σ_n^2, τ, and $\boldsymbol{\alpha}^T = (\alpha_1, \ldots, \alpha_K)$.

• USM: the case of interest in this contribution, where α_k is assumed to be random, zero mean complex white Gaussian distributed. Then,

$$\alpha_k \sim \mathcal{CN}(0, \sigma_\alpha^2), \quad (4a)$$

$$\mathbf{y}_k \sim \mathcal{CN}(\mathbf{0}, \mathbf{C}), \; \mathbf{C} = \sigma_\alpha^2 \mathbf{s}(\tau)\mathbf{s}^H(\tau) + \sigma_n^2 \mathbf{I}_N, \quad (4b)$$

and the unknown parameters are σ_n^2, τ and σ_α^2.

3. CONDITIONAL AND UNCONDITIONAL ML ESTIMATORS

In this section, the MLE expressions are given for both CSM and USM, which will be used to validate the CRBs derived in Sec. 4. For the rest of this study, $\|\mathbf{s}\|^2 \triangleq \mathbf{s}^H\mathbf{s}$, and the correlation between transmitted and received signals is $r_k(\tau) = \mathbf{s}^H(\tau)\mathbf{y}_k$, which is maximum at the true delay when $\mathbf{n}_k = \mathbf{0}$.

3.1. Conditional MLE

Given the CSM in (3), the CMLE consists in minimizing the negative log-likelihood function corresponding to (3) which can be recast as (subscript c for conditional) [20, Sec. 4.3.2],

$$\widehat{\tau}_c = \arg\max_{\tau} \sum_{k=1}^{K} |r_k(\tau)|^2, \quad (5)$$

$$\widehat{\alpha}_{k,c} = \frac{r_k(\widehat{\tau}_c)}{\|\mathbf{s}\|^2}, \quad (6)$$

$$\widehat{\sigma}_{n,c}^2 = \frac{1}{KN} \sum_{k=1}^{K} \left(\|\mathbf{y}_k\|^2 - \frac{|r_k(\widehat{\tau}_c)|^2}{\|\mathbf{s}\|^2} \right). \quad (7)$$

In addition, because the estimation of σ_α^2 is of particular interest and it is not directly available through the CMLE, the unbiased variance estimator obtained from the individual $\widehat{\alpha}_{k,c}$ is also considered,

$$\widehat{\sigma}_{\alpha,c}^2 = \frac{1}{K-1} \sum_{k=1}^{K} \left(\widehat{\alpha}_{k,c} - \frac{1}{K} \sum_{m=1}^{K} \widehat{\alpha}_{m,c} \right)^2. \quad (8)$$

3.2. Unconditional MLE

Given the USM in (4b), the criterion to minimize is the following negative log-likelihood,

$$L(\sigma_n^2, \tau, \sigma_\alpha^2) = \ln(|\mathbf{C}|) + \mathrm{Tr}\left\{ \mathbf{C}^{-1} \widehat{\mathbf{R}}_y \right\}, \quad (9)$$

where $\widehat{\mathbf{R}}_y = \frac{1}{K}\sum_{k=1}^{K} \mathbf{y}_k \mathbf{y}_k^H$ is the sample covariance matrix. It can be shown (see App. A for details) that this minimization leads to (subscript u for unconditional),

$$\widehat{\tau}_u = \arg\min_{\tau} \{C_u(\tau)\}, \quad (10)$$

$$C_u(\tau) = \left(\sum_{k=1}^{K} |r_k(\tau)|^2 \right) \left(\sum_{k=1}^{K} \|\mathbf{y}_k\|^2 - \frac{|r_k(\tau)|^2}{\|\mathbf{s}\|^2} \right)^{N-1},$$

$$\widehat{\sigma}_{\alpha,u}^2 = \frac{1}{K(N-1)\|\mathbf{s}\|^2} \sum_{k=1}^{K} N \frac{|r_k(\widehat{\tau}_u)|^2}{\|\mathbf{s}\|^2} - \|\mathbf{y}_k\|^2, \quad (11)$$

$$\widehat{\sigma}_{n,u}^2 = \frac{1}{K(N-1)} \sum_{k=1}^{K} \|\mathbf{y}_k\|^2 - \frac{|r_k(\widehat{\tau}_u)|^2}{\|\mathbf{s}\|^2}. \quad (12)$$

Notice that for a finite number of snapshots and in the high SNR regime $\widehat{\tau}_u$ is known to be non-efficient. In contrast, $\widehat{\tau}_u$ is efficient for a fixed SNR in the large snapshots regime. In addition, at high SNR, both CMLE and UMLE criteria are equivalent, then $\widehat{\tau}_c$ and $\widehat{\tau}_u$ provide the same estimates [17].

4. CLOSED-FORM UNCONDITIONAL CRB

Given the USM in (4b), the corresponding CRB_u for the estimation of $\boldsymbol{\theta}^T = (\tau, \sigma_\alpha^2, \sigma_n^2)$ is now derived. For complex Gaussian observations, the Fisher information matrix (FIM) entries can be obtained from the Slepian–Bangs formula (i.e., zero mean) [8],

$$[\mathbf{F}(\boldsymbol{\theta})]_{k,l} = K\,\mathrm{Tr}\left(\mathbf{C}^{-1} \frac{\partial \mathbf{C}}{\partial \theta_k} \mathbf{C}^{-1} \frac{\partial \mathbf{C}}{\partial \theta_l} \right), \quad (13)$$

It can be shown (see Appendix B for details) that the FIM takes the following form:

$$\mathbf{F}(\boldsymbol{\theta}) = K \begin{bmatrix} w_\tau & 0 & 0 \\ 0 & w_\alpha & w_{\alpha,n} \\ 0 & w_{\alpha,n} & w_n \end{bmatrix}, \quad (14)$$

with,

$$w_\tau = 2\frac{\beta^2}{1+\beta}\left(\frac{W_{33}}{w_1} - \frac{|w_3|^2}{w_1^2}\right),\ w_\alpha = \frac{\beta^2}{(\sigma_\alpha^2)^2(1+\beta)^2}, \tag{15}$$

$$w_n = \frac{N}{(\sigma_n^2)^2} - \frac{\beta(2+\beta)}{(\sigma_n^2)^2(1+\beta)^2},\ w_{\alpha,n} = \frac{\beta}{\sigma_n^2\sigma_\alpha^2(1+\beta)^2}, \tag{16}$$

where $\beta = \sigma_\alpha^2\|\mathbf{s}\|^2/\sigma_n^2$, w_1, w_3 and W_{33} are defined in App. B. Consequently, a closed-form expression of CRB_u for the estimation of $\boldsymbol{\theta}$ can be obtained by inverting $\mathbf{F}(\boldsymbol{\theta})$:

$$\mathrm{CRB}_u(\tau) = \frac{1+\beta}{2K\beta^2}\frac{1}{\frac{W_{33}}{w_1} - \frac{|w_3|^2}{w_1^2}}, \tag{17}$$

$$\mathrm{CRB}_u(\sigma_\alpha^2) = \frac{\left(\sigma_\alpha^2\right)^2\left(1+(N-1)\left(1+\beta\right)^2\right)}{K(N-1)\beta^2} \tag{18}$$

$$\mathrm{CRB}_u(\sigma_n^2) = \frac{(\sigma_n^2)^2}{K(N-1)}. \tag{19}$$

For completeness, the conditional CRB for the estimation of τ from the CSM (3) is recalled hereafter

$$\mathrm{CRB}_c(\tau) = \frac{1}{2K\gamma}\frac{1}{\frac{W_{33}}{w_1} - \frac{|w_3|^2}{w_1^2}}, \tag{20}$$

where $\gamma = \frac{1}{K}\sum_{k=1}^{K}|\alpha_k|^2\|\mathbf{s}\|^2/\sigma_n^2$. Notice that for a large number of snapshots K, γ tends to β, highlighting the fact that $\mathrm{CRB}_u(\tau) \geq \mathrm{CRB}_c(\tau)$, with equality in the large number of snapshots and/or high SNR regimes.

5. VALIDATION

In order to validate the proposed CRBs, and assess the performance of the corresponding ML estimators, two representative band-limited signals are considered: 1) a GPS L1 C/A signal (sampled at 4 MHz), and 2) a radar chirp signal (bandwidth 250 kHz). The root MSE (RMSE) is computed over 5000 Monte Carlo runs, for the estimation of σ_α^2 and τ. The RMSE results for σ_α^2, with respect to the number of snapshots, are shown in Fig. 1 (for a fixed SNR = -22dB, for a single snapshot, at the input of the receiver). For both signals one can clearly see that: i) the UMLE outperforms the CMLE at low number of snapshots, and ii) while the UMLE is efficient, the CMLE is not. This result validates the proposed CRB expression and shows the interest in considering the USM to extract the energy information from a reflecting surface. Notice that while the UMLE provides the optimal solution, the artificial sample variance obtained from individual CMLE α_k estimates is suboptimal in the MSE sense.

The results for the second parameter of interest, τ, are shown in Fig. 2 (again for a fixed SNR=-22dB). The global behavior is again the same for both signals. As predicted by

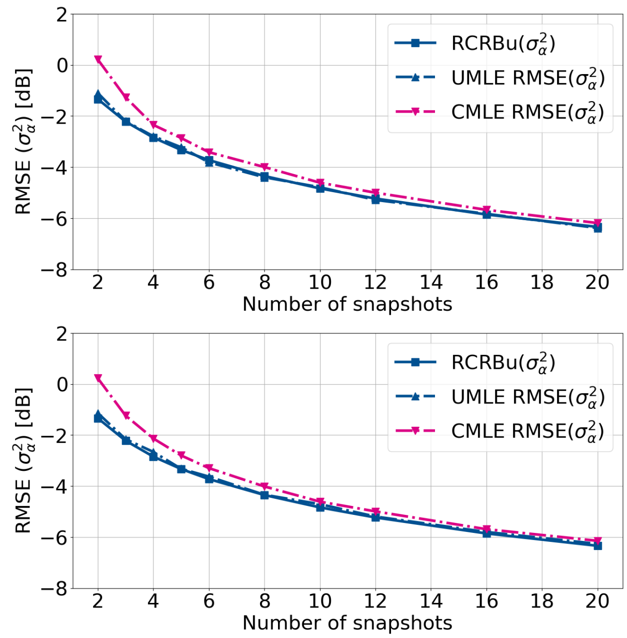

Fig. 1: RMSE of the variance estimate $\widehat{\sigma}_\alpha^2$ for both UMLE and CMLE, together with the root CRB_u. (Top) GPS signal; (Bottom) Chirp signal.

the theory [17]: i) $\mathrm{CRB}_u(\tau)$ is slightly larger than $\mathrm{CRB}_c(\tau)$ (which was evaluated for $\gamma = \beta$ in (20)), even if the difference in this scenario is marginal; ii) the performance of both UMLE and CMLE are almost equivalent; iii) both estimators are not efficient at a low number of snapshots, with a performance that gets close to the CRB after a minimum number of snapshots; iv) both estimators approach the CRB as the number of snapshots increases. Again, this validates both the proposed CRB and MLE expressions.

Finally, to ensure that for a rather large number of snapshots and an increasing SNR the performance of both UMLE and CMLE converge, the RMSE of the estimation of τ is computed for a fixed number of snapshot ($K = 20$) and varying SNR. The results are shown in Fig. 3. In such conditions (large SNR and K), both estimators have the same behavior and the RMSE reaches the computed CRB (which are equal in this regime). This last results complete the validation of the expression derived in Sec. 4.

6. CONCLUSIONS

In this contribution, the use of an USM for the estimation of both a propagation delay and a Gaussian source variance (i.e., transmitter-to-receiver propagation path going through a reflection) was explored. The application that motivated this study is GNSS-based reflectometry, where the main interest is characterizing the energy of the reflecting surface to build surface maps as done in airborne radar (SAR) [14, 15]), but the standard processing does not take into account a possible random scattering [6]. The unconditional CRB for

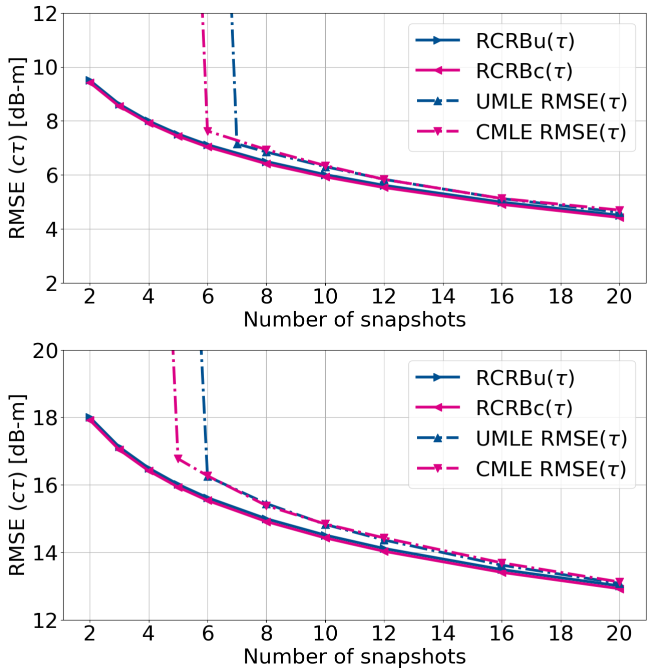

Fig. 2: RMSE of the delay estimate $\hat{\tau}$ for both UMLE and CMLE, together with the root CRB_u and CRB_c. (Top) GPS signal; (Bottom) Chirp signal.

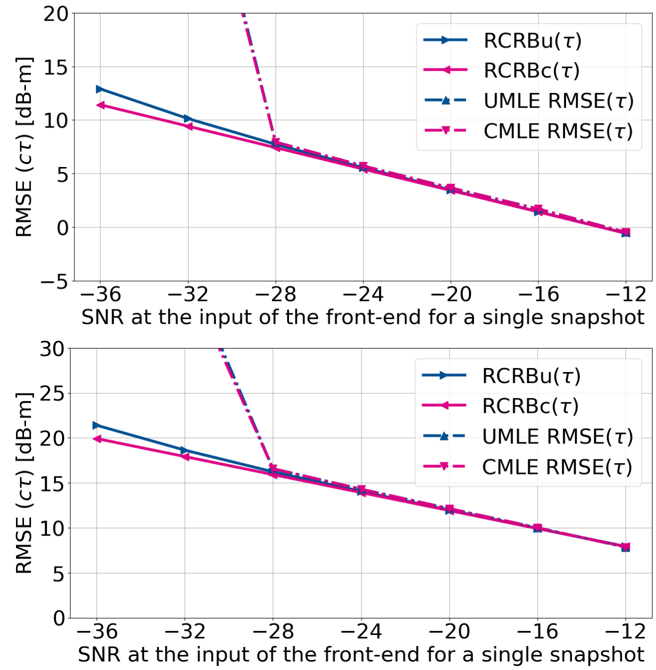

Fig. 3: RMSE of the delay estimate $\hat{\tau}$ for both UMLE and CMLE, together with the root CRB_u and CRB_c as a function of the SNR, for $K = 20$ snapshots. (Top) GPS signal; (Bottom) Chirp signal.

the parameters of interest was derived, and the corresponding ML expressions implemented. Numerical results were provided with two representative signals, first to validate the proposed CRB, and second to assess the performance of the corresponding ML estimator. For the estimation of the Gaussian source variance, while the UMLE is efficient, the CMLE is not. For the delay estimation, both estimators provide the same performance.

A. DETAILS ON UML ESTIMATORS

Dropping the dependence on τ for the sake of clarity, $\mathbf{C} = \sigma_\alpha^2 \mathbf{s}\mathbf{s}^H + \sigma_n^2 \mathbf{I}_N$, and its determinant and inverse can be expressed as,

$$|\mathbf{C}| = \left(\sigma_n^2\right)^N \left| \frac{\sigma_\alpha^2}{\sigma_n^2} \mathbf{s}\mathbf{s}^H + \mathbf{I}_N \right| = \left(\sigma_n^2\right)^{N-1} \left(\sigma_\alpha^2 \|\mathbf{s}\|^2 + \sigma_n^2\right), \tag{21}$$

$$\mathbf{C}^{-1} = \frac{1}{\sigma_n^2} \left(\mathbf{I}_N - \frac{\sigma_\alpha^2}{\sigma_\alpha^2 \|\mathbf{s}\|^2 + \sigma_n^2} \right). \tag{22}$$

Using (21) and (22), the criterion (9) can be written as,

$$L(\sigma_n^2, \tau, \sigma_\alpha^2) = (N-1)\ln(\sigma_n^2) + \ln\left(\sigma_n^2 + \sigma_\alpha^2 \|\mathbf{s}\|^2\right) + \frac{1}{K}\sum_{k=1}^{K} \frac{\|\mathbf{y}_k\|^2}{\sigma_n^2} - \frac{1}{K}\sum_{k=1}^{K} \frac{\sigma_\alpha^2 |r_k(\tau)|^2}{\sigma_n^2\left(\sigma_n^2 + \sigma_\alpha^2 \|\mathbf{s}\|^2\right)}. \tag{23}$$

The results presented in Sec. 3.2 are then obtained by zeroing the derivatives of the criterion first with respect to σ_n^2 and σ_α^2.

B. DETAILS ON THE UNCONDITIONAL CRB

In this section, details are given on the evaluation of the FIM terms (14). First the derivatives of the covariance matrix $\mathbf{C}$ are evaluated, with the time derivative of $\mathbf{s}$: $\dot{\mathbf{s}} = \mathrm{d}\mathbf{s}/\mathrm{d}t$,

$$\frac{\partial \mathbf{C}}{\partial \sigma_\alpha^2} = \mathbf{s}\mathbf{s}^H, \frac{\partial \mathbf{C}}{\partial \sigma_n^2} = \mathbf{I}_N, \frac{\partial \mathbf{C}}{\partial \tau} = -\sigma_\alpha^2 \left(\dot{\mathbf{s}}\mathbf{s}^H + \mathbf{s}\dot{\mathbf{s}}^H\right). \tag{24}$$

From (24), the following intermediate terms comes directly:

$$\mathbf{C}^{-1}\frac{\partial \mathbf{C}}{\partial \tau} = -\frac{\sigma_\alpha^2}{\sigma_n^2}\left(\dot{\mathbf{s}}\mathbf{s}^H + \frac{\mathbf{s}\dot{\mathbf{s}}^H}{1+\beta} - \frac{w_3 \beta \mathbf{s}\mathbf{s}^H}{w_1(1+\beta)}\right) \tag{25}$$

$$\mathbf{C}^{-1}\frac{\partial \mathbf{C}}{\partial \sigma_\alpha^2} = \frac{1}{\sigma_n^2(1+\beta)}\mathbf{s}\mathbf{s}^H \tag{26}$$

$$\mathbf{C}^{-1}\frac{\partial \mathbf{C}}{\partial \sigma_n^2} = \frac{1}{\sigma_n^2}\left(\mathbf{I}_N - \frac{\sigma_\alpha^2}{\sigma_n^2(1+\beta)}\mathbf{s}\mathbf{s}^H\right) \tag{27}$$

where $w_1 = \mathbf{s}^H\mathbf{s}/F_s$, $w_3 = \mathbf{s}^H\dot{\mathbf{s}} = \mathbf{s}^H\mathbf{\Lambda}\mathbf{s}/F_s$, $W_{33} = \dot{\mathbf{s}}^H\dot{\mathbf{s}}/F_s = F_s\mathbf{s}^H\mathbf{V}\mathbf{s}$ were introduced in [9]. The definition of $\mathbf{\Lambda}$ and $\mathbf{V}$ are recalled hereafter:

$$[\mathbf{\Lambda}]_{k,l} = \begin{cases} 0 & \text{if } k = l, \\ \frac{(-1)^{|k-l|}}{k-l} & \text{else.} \end{cases} \tag{28}$$

$$[\mathbf{V}]_{k,l} = \begin{cases} \pi^2/3 & \text{if } k = l, \\ \frac{2(-1)^{|k-l|}}{(k-l)^2} & \text{else.} \end{cases} \tag{29}$$

The final results are obtained by evaluating the trace of the cross products between each three terms from (25)–(27).

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