# Deliberate model misspecification for weather radar signal Doppler spectrum moments estimation


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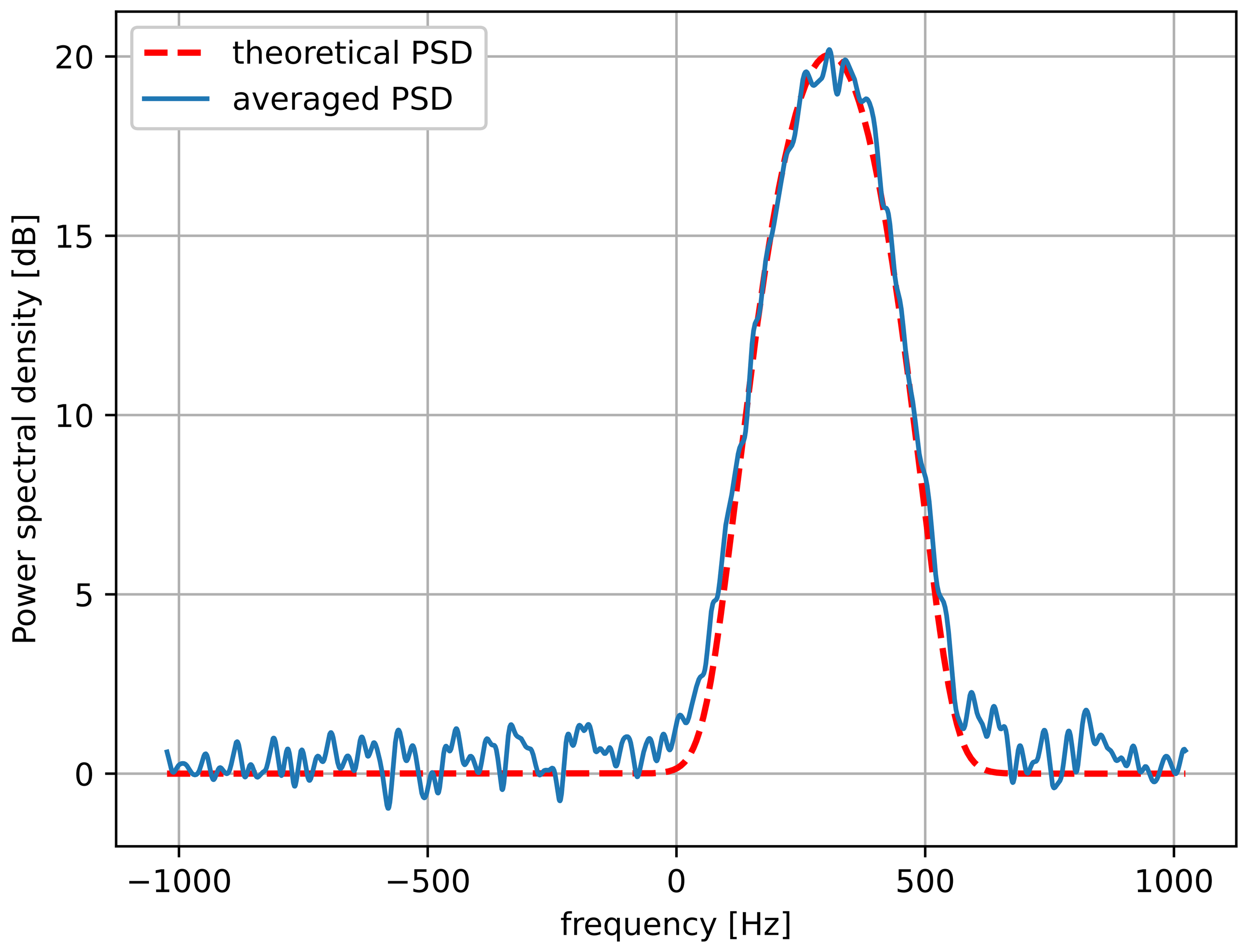
## Context

In weather applications, ground based pulsed radars are used to estimate precipitation rates. The main features (moments of the Doppler spectrum) of the collected signals are the following:

- moment 0: mean power $P_0 \rightarrow$ reflectivity,
- moment 1: mean Doppler frequency $f_0 \rightarrow$ mean radial velocity of raindrops,
- moment 2: spectral width $\sigma_0 \rightarrow$ dispersion of radial velocities (seldom used).




## Misspecification estimation

There is model misspecification when the assumed signal model (based on which an estimator is designed) differs from the true signal model.

- imperfect knowledge of the true data model,
- true data model known but the corresponding estimation problem is too involved.

## Signal models

A general signal model for the problem at hand is the following:

$$\mathbf{y} = \mathbf{x} + \mathbf{w},$$

with $\mathbf{w} \sim \mathcal{CN}\left(\mathbf{0}, \sigma_n^2 \mathbf{I}_N\right)$, an additive white complex Gaussian noise of power $\sigma_n^2$ and $\mathbf{x}$ the signal of interest.

### True signal model

The shape of the weather signals power spectrum is assumed Gaussian.

$$\mathbf{x} \sim \mathcal{CN}\left(\mathbf{0}, \mathbf{B}_t(P_0, f_0, \sigma_0^2)\right),$$

where,

$$\mathbf{B}_t(P_0, f_0, \sigma_0^2) = P_0 \mathbf{D}(f_0)\mathbf{C}(\sigma_0^2)\mathbf{D}(f_0)^H,$$
$$\mathbf{D}(f_0) = \text{diag}\left(\ldots, \exp\left(j2\pi f_0(n-1)T_s\right), \ldots\right),$$
$$\left[\mathbf{C}(\sigma_0^2)\right]_{k,l} = \exp\left(-2\pi^2\sigma_0^2(k-l)^2T_s^2\right).$$

### Assumed signal model

One may ignore the presence of the spectral width $\sigma_0$. The resulting power spectrum would shrink to a dirac function centered on $f_0$.

$$\mathbf{x} \sim \mathcal{CN}\left(\mathbf{0}, \mathbf{B}_a(P_0, f_0)\right),$$

where

$$\mathbf{B}_a(P_0, f_0) = P_0 \mathbf{d}(f_0)\mathbf{d}(f_0)^H,$$
$$\mathbf{d}(f_0) = \left(\ldots, \exp\left(j2\pi f_0(n-1)T_s\right), \ldots\right).$$

## Considered estimators

### Mismatched unconditional maximum likelihood estimator

$$L(\boldsymbol{\mu}) = \ln\left(|\mathbf{R}(\boldsymbol{\mu})|\right) + \text{Tr}\left\{\mathbf{R}(\boldsymbol{\mu})^{-1}\widehat{\mathbf{R}}_{\mathbf{y}}\right\},$$

with the sample covariance matrix $\widehat{\mathbf{R}}_{\mathbf{y}} = \frac{1}{K}\sum_{k=1}^{K} \mathbf{y}_k \mathbf{y}_k^H$ and the asymptotic covariance matrix $\mathbf{R}(\boldsymbol{\mu}) = \mathbf{B}_a(P_0, f_0) + \sigma_n^2 \mathbf{I}_N$. Then,

$$\widehat{f}_0 = \arg\max_f \sum_{k=1}^{K} |r_k(f)|^2, \quad \widehat{P}_0 = \frac{1}{KN(N-1)} \sum_{k=1}^{K} \left(|r_k(\widehat{f}_0)|^2 - \|\mathbf{y}_k\|^2\right),$$

where $r_k(f) = \mathbf{y}_k^H \mathbf{d}(f)$.

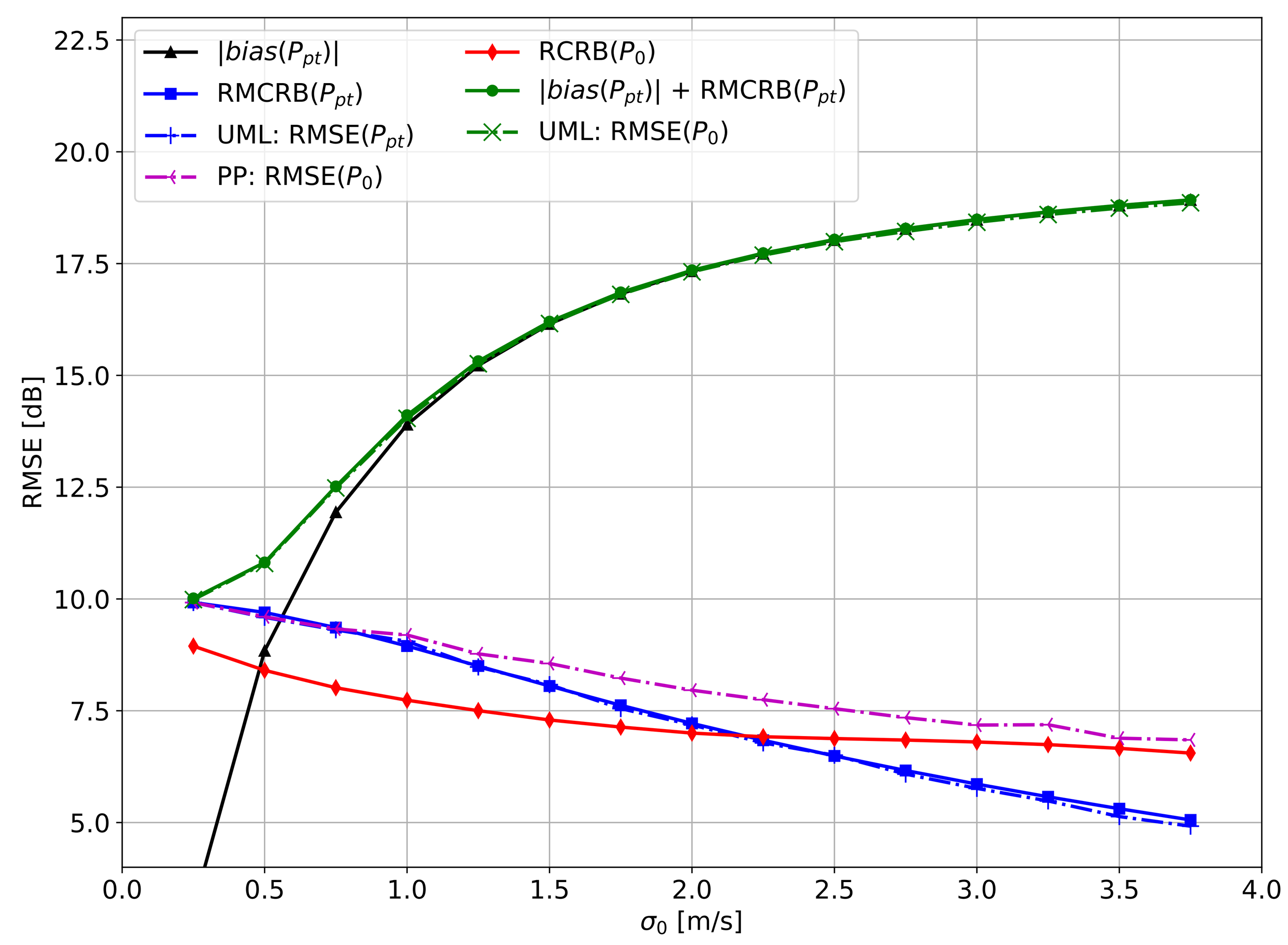
### Pulse-pair estimator

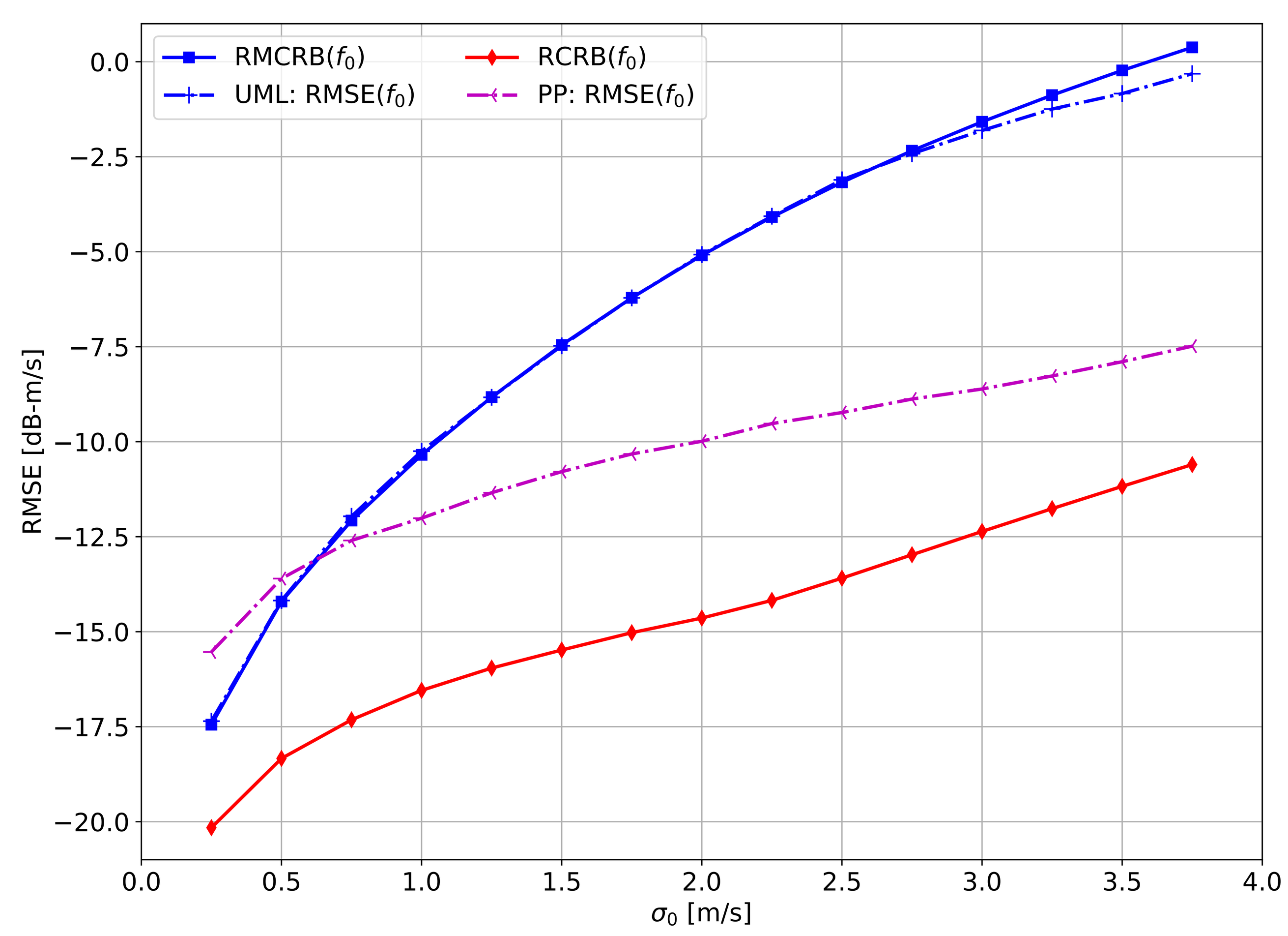
Wdely used in operational systems, it requires an estimate of the auto-correlation.

$$\widehat{A}_k(nT_s) = \frac{1}{N-|n|} \sum_{i=0}^{N-1} [\mathbf{y}_k]_i^* [\mathbf{y}_k]_{i+n}.$$

$$\widehat{P}_{PP} = \frac{1}{K}\sum_{k=1}^{K} \widehat{A}_k(0), \quad \widehat{f}_{PP} = \frac{1}{2\pi T_s} \frac{1}{K} \sum_{k=1}^{K} \arg\left(\widehat{A}_k(T_s)\right).$$

## Results




## Conclusions and perspectives

- ignoring $\sigma_0$ may be interesting for a limited range of spectral width value,
- the approach may be applied to other misspecifications: spectrum shape, noise,
- operational constraints should be accounted for: pulse repetition time