Towards surface mapping using GNSS-IR

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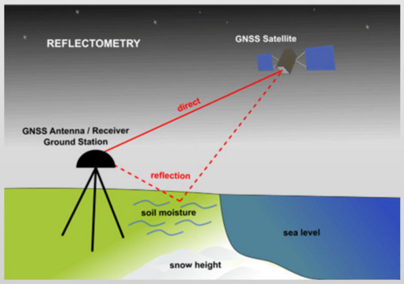
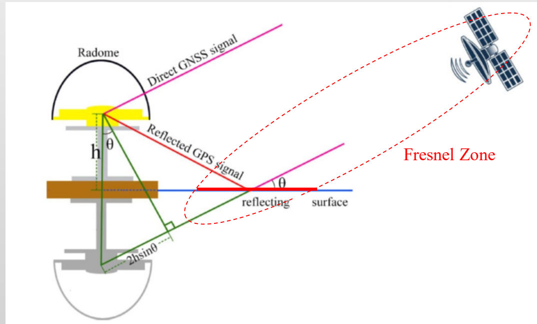
Framework – State of the Art

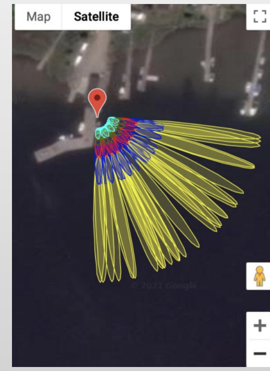
Global Navigation Satellites Systems main goal → Location and Timing

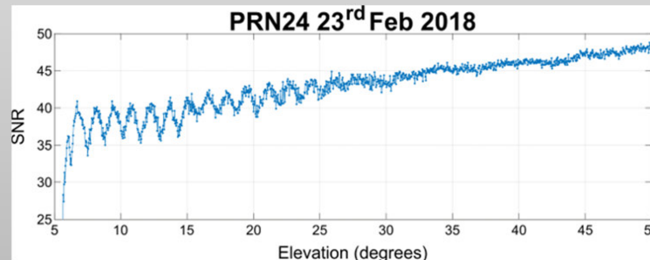
Multipath → GNSS-R

GNSS-R with ground receiver → GNSS-IR









→ **Hypothesis** : flat ground surface with a single specular reflection

$$r(t) = \beta e^{2i\pi f t}\left[c(t-\tau) + \alpha c(t-\tau-\Delta\tau) e^{-2i\pi \frac{2h\sin\theta}{\lambda}}\right] + n(t)$$

→ $$C/N_0 = \left|\beta\left(1 + \alpha e^{-2i\pi \frac{2h\sin\theta}{\lambda}}\right)\right| + n(t)$$

→ Information about h and α

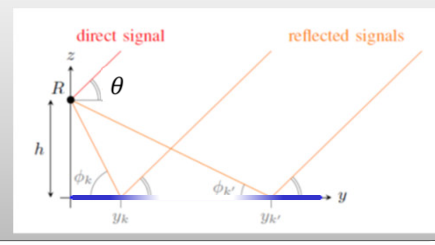
Signal model improvement

→ $$y(t) = \beta\left[1 + \sum \alpha_k e^{\frac{-2i\pi h}{\lambda}\left[\sin\theta(t) + \frac{1-\cos\varphi_k\cos\theta(t)}{\sin\varphi_k}\right]}\right]$$ for each satellite l in view

→ $\boldsymbol{y_l} = \beta_l \boldsymbol{M_l}\boldsymbol{\alpha} + \boldsymbol{n_l}$ for $l = 1, \dots, L$ → $\hat{\boldsymbol{\alpha}}$

where $\boldsymbol{M_l} = [\boldsymbol{1_N} \quad \boldsymbol{m}_l(1) \quad \dots \quad \boldsymbol{m}_l(K)]$ $\quad \boldsymbol{\alpha} = [1 \quad \alpha_1 \quad \dots \quad \alpha_K]^{\boldsymbol{T}}$

$$\boldsymbol{m}_l(k) = \left[e^{\frac{-2i\pi h}{\lambda}\left[\sin\theta(t_1) + \frac{1-\cos\varphi_k\cos\theta(t_1)}{\sin\varphi_k}\right]} \quad \dots \quad e^{\frac{-2i\pi h}{\lambda}\left[\sin\theta(t_N) + \frac{1-\cos\varphi_k\cos\theta(t_N)}{\sin\varphi_k}\right]}\right]^T$$



Asymptotically efficient solution

→ EXtended Invariance Principle (EXIP)

→ Reparameterization: $\boldsymbol{y_l} = \boldsymbol{M_l}\boldsymbol{\gamma_l} + \boldsymbol{n_l}$ → $\hat{\boldsymbol{\gamma}}_l = Argmin\, L_l(\boldsymbol{\gamma_l}) = Argmin\|\boldsymbol{y_l} - \boldsymbol{M_l}\boldsymbol{\gamma_l}\|^2$ → $\hat{\boldsymbol{\gamma}}_l = \left(\boldsymbol{M_l^H}\boldsymbol{M_l}\right)^{-1}\boldsymbol{M_l^H}\boldsymbol{y_l}$

→ Refinement: $\hat{\boldsymbol{\alpha}}, \hat{\boldsymbol{\beta}} = argmin \sum_{l=1}^{L} \|\hat{\boldsymbol{y}}_l - \beta_l \boldsymbol{\alpha}\|^2_{Q_l}$ where $\boldsymbol{Q_l} = \frac{\partial^2 L_l}{\partial \boldsymbol{\gamma_l} \partial \boldsymbol{\gamma_l}^{\boldsymbol{H}}} \cong N\boldsymbol{I}$ → $\hat{\boldsymbol{\alpha}}, \hat{\boldsymbol{\beta}} = argminTr[(\boldsymbol{\Gamma} - \boldsymbol{\alpha}\boldsymbol{\beta}^T)^H(\boldsymbol{\Gamma} - \boldsymbol{\alpha}\boldsymbol{\beta}^T)]$

→ $\hat{\boldsymbol{\alpha}}$ is the eigenvector associated with the larger eigenvalue of $\boldsymbol{\Gamma}\boldsymbol{\Gamma}^{\boldsymbol{H}}$ with $\hat{\boldsymbol{\alpha}}(1) = 1$ where $\boldsymbol{\Gamma} = [\hat{\boldsymbol{\gamma}}_1 \quad \dots \quad \hat{\boldsymbol{\gamma}}_L]$

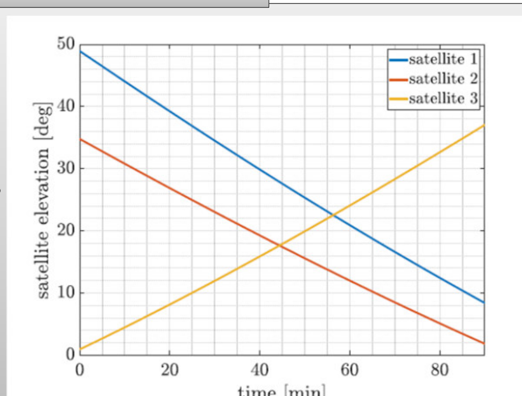
Numerical illustrations

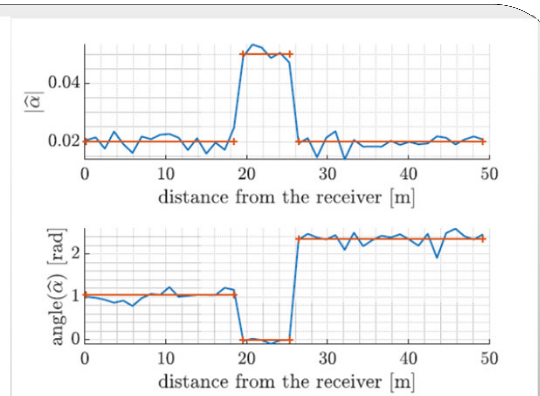
→ h = 2 m.

→ 3 satellites observed during 1 hour and half.

→ Satellites SNRs [15,5 17,5 20]dB

→ 3 different areas on the ground





Conclusions

→ Extended the standard specular GNSS-IR model to provide more precise mapping description.

→ Proposed an asymptotically efficient solution for the ground reflection coefficients estimation.