TéSA Seminar

Signal Processing for GNSS-R

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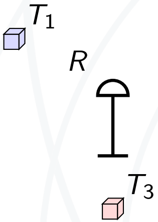
February 8, 2022

Outline

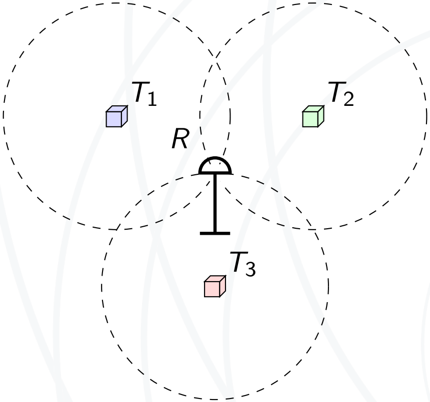
Outline

Global Navigation Satellite System (GNSS)



- constellations (GPS, GALILEO, etc),

Global Navigation Satellite System (GNSS)



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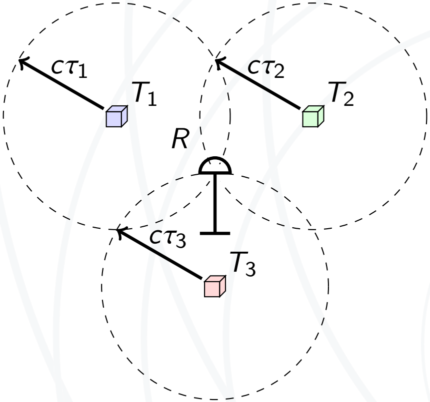
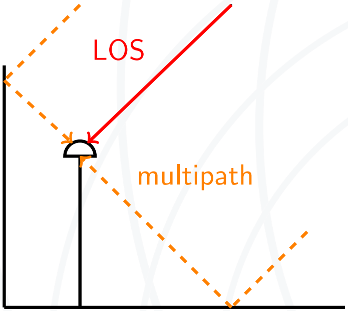


- constellations (GPS, GALILEO, etc),
- known signals (PRN),
- signal propagation,
- positioning by trilateration.

The Multipath Problem

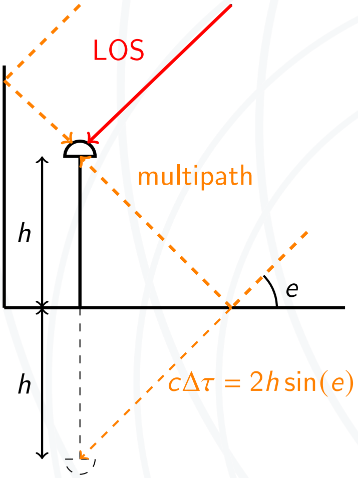


Definition*: *Multipath is the reception of multiple reflected or diffracted replicas of the desired signal, along with the direct path signal.*

- degradation of the estimation (bias induced),
- mobile application: random and dynamic phenomenon,

*[1] Kaplan and Hegarty, "Understanding GPS/GNSS: Principle and Applications," 2017.

The Multipath Problem



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- degradation of the estimation (bias induced),
- mobile application: random and dynamic phenomenon,
- it contains information!

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GNSS-Reflectometry

- GNSS signals: received 24/7 anywhere on Earth: signals of opportunity,
- Reflecting surfaces properties: remote sensing (altimetry, biomass, wind speed, soil moisture, etc.),

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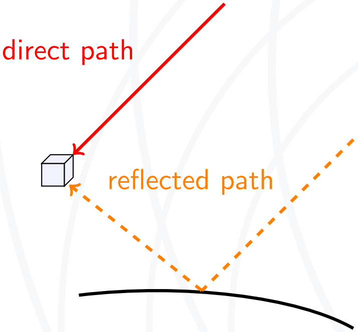
- Reflecting surfaces properties: remote sensing (altimetry, biomass, wind speed, soil moisture, etc.),
- GNSS-R: Study of GNSS signals reflections upon the Earth.

Spaceborne GNSS-R

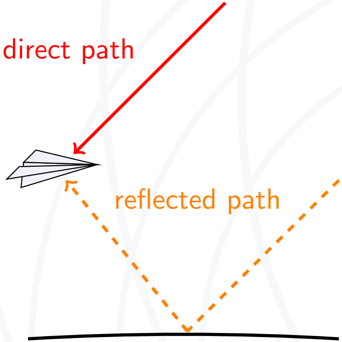


Spaceborne GNSS-R

- Low Earth Orbit satellites (CYGNSS, Hydro-GNSS),
- sea surface wind speed,
- important coverage and revisit time*,
- mixture of coherent and non-coherent reflection (scattering),
- resolution due to the satellite motion.

*[2] Zavorotny et al, "Tutorial on Remote Sensing Using GNSS Bistatic Radar of Opportunity," 2014.

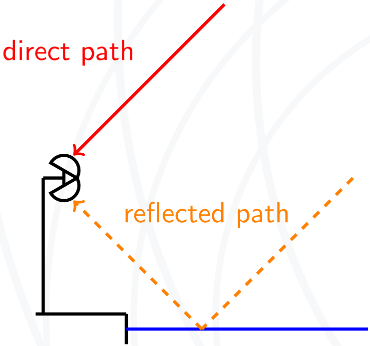
Airborne GNSS-R



Airborne GNSS-R

- various platforms: airplane*, UAV, etc.
- better quality of the reflected signal,
- sea level height, biomass,
- signal potentially more coherent,
- resolution due to the aircraft motion.

*[3] Ribó et al, "A Software-Defined GNSS Reflectometry Recording Receiver with Wide-Bandwidth Multi-Band Cabability and Digital beam-Forming," 2017.

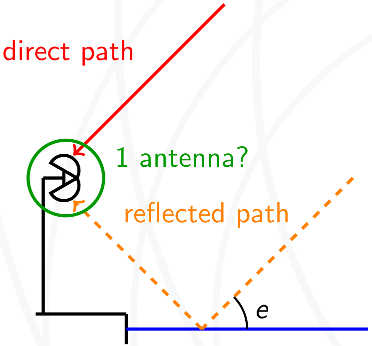


Ground-based GNSS-R

- coherent reflection,
- snow cover, soil moisture and tide monitoring,
- static installation, local coverage.
- 1 antenna: study on overall power only*.

*[4] Ribot et al, "Normalized GNSS Interference Pattern Technique for Altimetry," 2014.

Ground-Based GNSS-R



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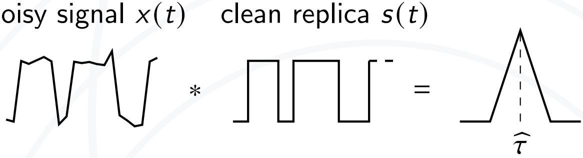
GNSS-R: Existing Techniques

- a word about correlation...



GNSS-R: Existing Techniques

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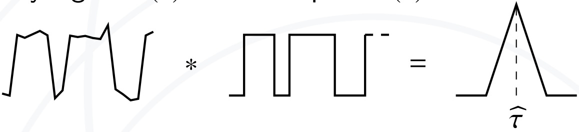
noisy signal $x(t)$ clean replica $s(t)$



- Conventional GNSS-R: convolution with a clean replica:
  - track of a chosen satellite signal,
  - limited to the known signals.

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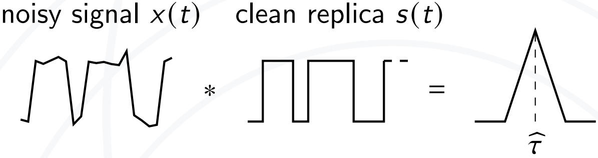

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- Interferometric GNSS-R: convolution between the direct and the reflected path:
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  - potential ambiguity between the different sources.

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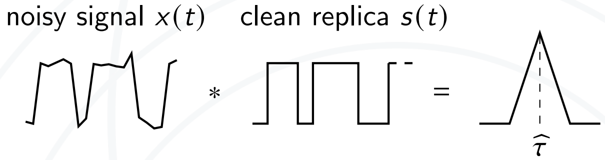

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- Ground-based: GNSS Interferometric Reflectometry:
  - interference between the signals (satellite elevation, height),
  - does not fully exploit the fact that the signals are known...

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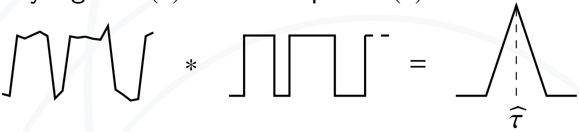

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Signal Model

- Dual source model with an assumed specular reflection:

$$\mathbf{x} = \mathbf{A}(\boldsymbol{\eta}_0, \boldsymbol{\eta}_1)\boldsymbol{\alpha} + \mathbf{w},\ \ \mathbf{w} \sim \mathcal{CN}(0, \sigma_n^2 \mathbf{I}_N), \tag{1}$$

with, for $\boldsymbol{\eta}^T = [\tau_, F_d]$,

$$\mathbf{A}(\boldsymbol{\eta}_0, \boldsymbol{\eta}_1) = \left[\mathbf{s}(\boldsymbol{\eta}_0),\, \mathbf{s}(\boldsymbol{\eta}_1)\right], \tag{2}$$

$$\mathbf{s}(\boldsymbol{\eta}) = \left(\ldots,\, s(nT_s - \tau)e^{-j2\pi F_d(nT_s-\tau)},\, \ldots\right), \tag{3}$$

$$\boldsymbol{\alpha}^T = \left(\rho_0 e^{j\phi_0},\, \rho_1, e^{j\phi_1}\right). \tag{4}$$

- Deterministic formulation with the following unknown vector:

$$\boldsymbol{\epsilon}^T = [\sigma_n^2, \underbrace{\tau_0, F_{d,0}, \rho_0, \phi_0}_{\boldsymbol{\theta}_0^T}, \underbrace{\tau_1, F_{d,1}, \rho_1, \phi_1}_{\boldsymbol{\theta}_1^T}] \tag{5}$$

Signal Model

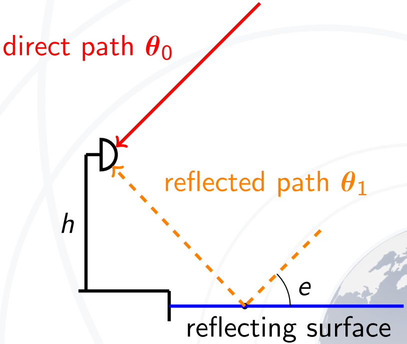

Figure: Geometry considered.

Cramér-Rao Bounds (CRB)

- Problem: estimate $\boldsymbol{\epsilon}$.
- Cramér-Rao bound: theoretical lower bound for the variance of any unbiased estimator,
- from the signal model, obtain the Fisher Information Matrix by using of the Slepian-Bangs formula*:

$$\left[\mathbf{F}_{\boldsymbol{\epsilon}|\boldsymbol{\epsilon}}(\boldsymbol{\epsilon})\right]_{k,l} = \frac{2}{\sigma_n^2}\mathrm{Re}\left\{\left(\frac{\partial \mathbf{A}\boldsymbol{\alpha}}{\partial \epsilon_k}\right)^H \left(\frac{\partial \mathbf{A}\boldsymbol{\alpha}}{\partial \epsilon_l}\right)\right\} + \frac{N}{\sigma_n^4}\frac{\partial \sigma_n^2}{\partial \epsilon_k}\frac{\partial \sigma_n^2}{\partial \epsilon_l}, \quad (6)$$

- the CRB for the estimation of $\boldsymbol{\epsilon}$ is obtained by inverting the FIM:

$$\mathbf{CRB}_{\boldsymbol{\epsilon}|\boldsymbol{\epsilon}}(\boldsymbol{\epsilon}) = \left[\mathbf{F}_{\boldsymbol{\epsilon}|\boldsymbol{\epsilon}}(\boldsymbol{\epsilon})\right]^{-1} \quad (7)$$

*[5] Yau and Bresler, "A Compact Cramér-Rao Bound Expression for Parametric Estimation of Superimposed Signals," 1992.

Cramér-Rao Bounds (CRB)

$$\mathbf{CRB}_{\boldsymbol{\epsilon}|\boldsymbol{\epsilon}}(\boldsymbol{\epsilon}) = \begin{bmatrix} F_{\sigma_n^2|\boldsymbol{\epsilon}}(\boldsymbol{\epsilon}) & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{F}_{\theta_0|\boldsymbol{\epsilon}}(\boldsymbol{\epsilon}) & \mathbf{F}_{\theta_0,\theta_1|\boldsymbol{\epsilon}}(\boldsymbol{\epsilon}) \\ \mathbf{0} & \mathbf{F}_{\theta_1,\theta_0|\boldsymbol{\epsilon}}(\boldsymbol{\epsilon}) & \mathbf{F}_{\theta_1|\boldsymbol{\epsilon}}(\boldsymbol{\epsilon}) \end{bmatrix}^{-1} \tag{8}$$

- closed-form expression in terms of the signal baseband samples,
- $\mathbf{F}_{\theta_i|\boldsymbol{\epsilon}}(\boldsymbol{\epsilon})$: known uncoupled contribution from each signal,
- $\mathbf{F}_{\theta_1,\theta_0|\boldsymbol{\epsilon}}(\boldsymbol{\epsilon})$: interference term*!

*[6] Lubeigt et al, "Joint Delay-Doppler Estimation Performance in a Dual Source Context," 2020.

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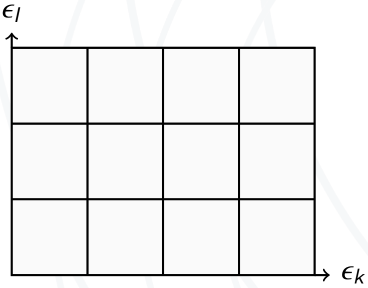
- To validate this expression: implementation of an efficient (unbiased and variance equal to the CRB) estimator and check its variance!

*[6] Lubeigt et al, "Joint Delay-Doppler Estimation Performance in a Dual Source Context," 2020.

Validation Method: The Dual Source Maximum Likelihood Estimator (2S-MLE)

- Property of the 2S-MLE: asymptotically efficient*.
- Implementation: 4 dimensional search...



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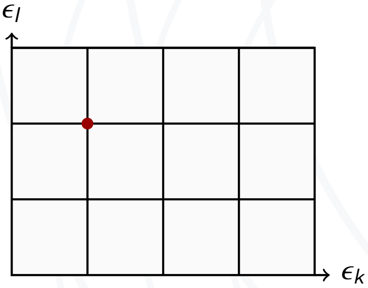

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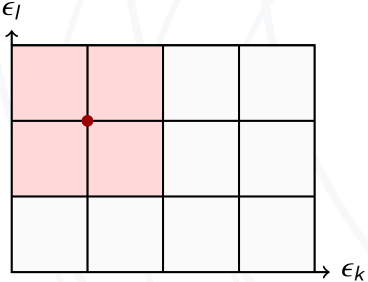

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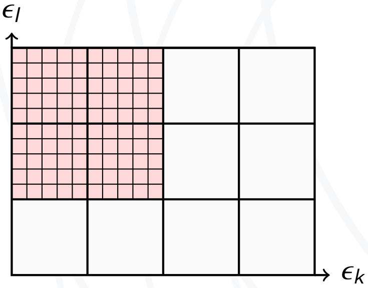

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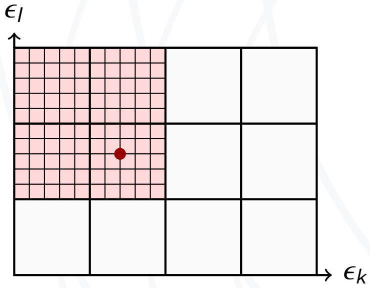

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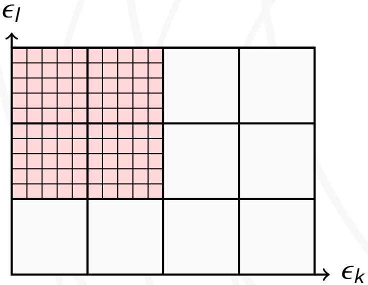

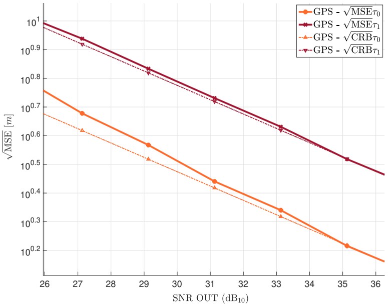

Figure: RMSE of the 2S-MLE $\widehat{\tau_0}$ and $\widehat{\tau_1}$ along with corresponding $\sqrt{\text{CRB}}$.

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Outline

Potential Algorithms

- Estimators based on the 2S-MLE (but less complex),
- existing algorithms from the GNSS community (multipath mitigation): CLEAN-RELAX Estimator (MEDLL)[*],
- or from the radar community: Alternating Projection Estimator[†].

[*][8] Van Nee, "The Multipath Estimating Delay Lock Loop," 1992.

[†][9] Ziskind and Wax, "Maximum Likelihood Localization Multiple Sources by Alternating Projection," 1988.

CLEAN-RELAX Estimator

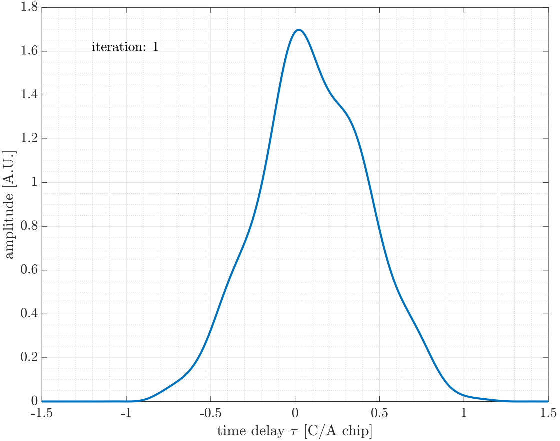

Figure: First estimation.

CLEAN-RELAX Estimator

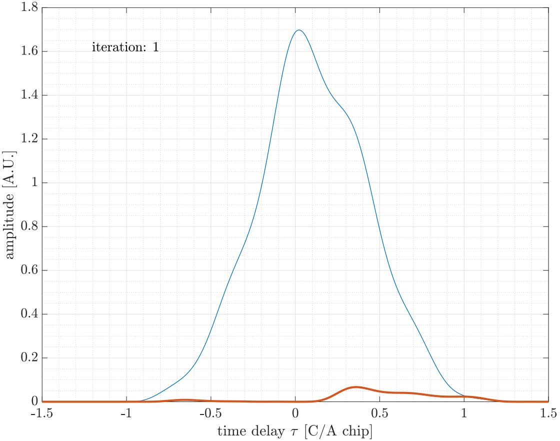

Figure: Second estimation upon the residue.

CLEAN-RELAX Estimator

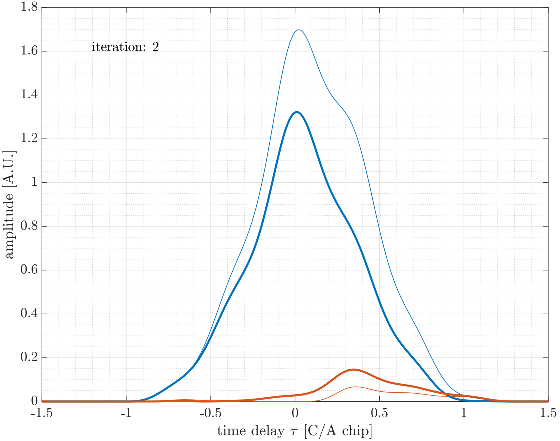

Figure: Iterate...

CLEAN-RELAX Estimator

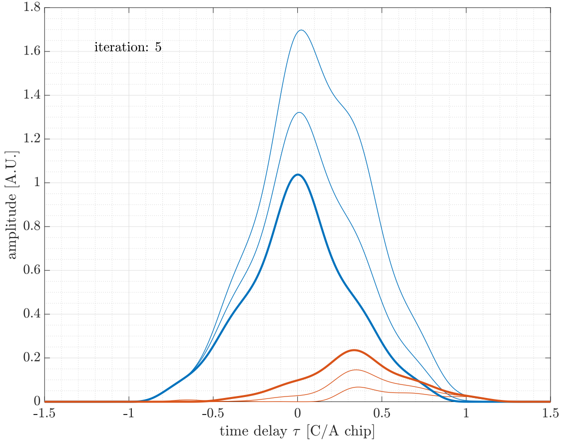

Figure: ... until convergence.

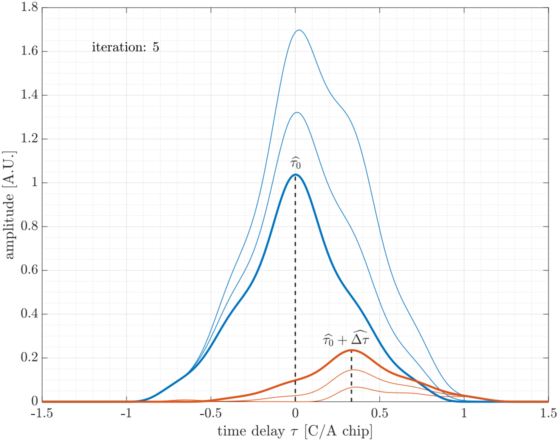

Figure: Read the estimates.

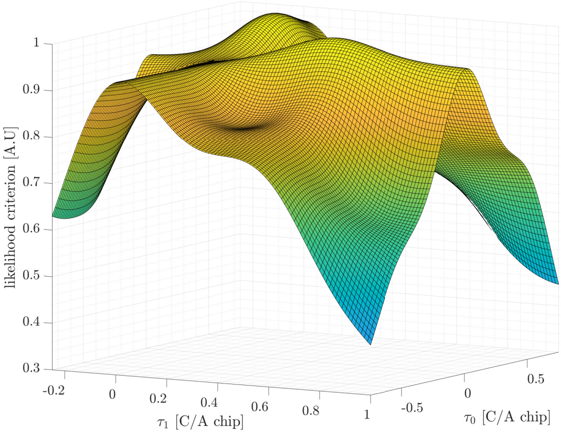

Figure: Likelihood function to be maximized.

Alternating Projector Estimator

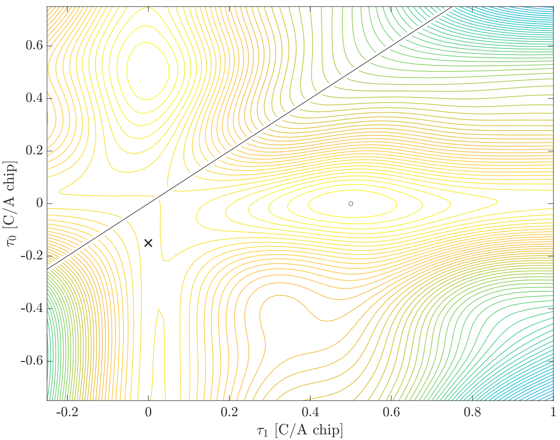

Figure: Initialization.

Alternating Projector Estimator

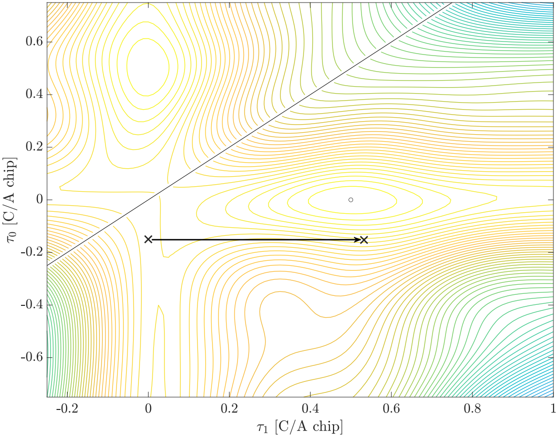

Figure: Maximize w.r.t. τ_1 for τ_0 fixed...

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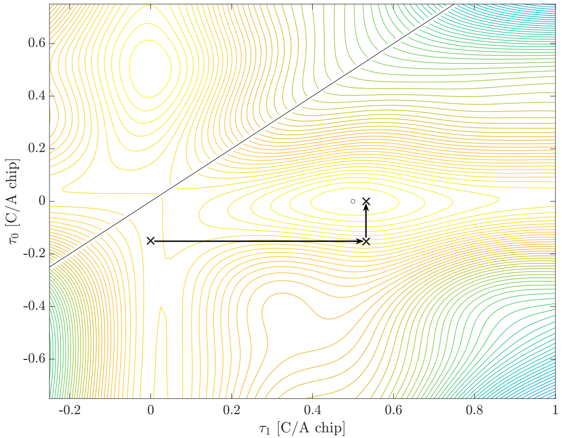
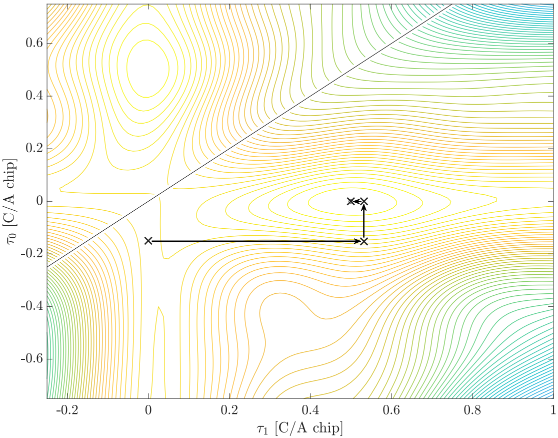


Figure: ... and vice-versa...

Alternating Projector Estimator



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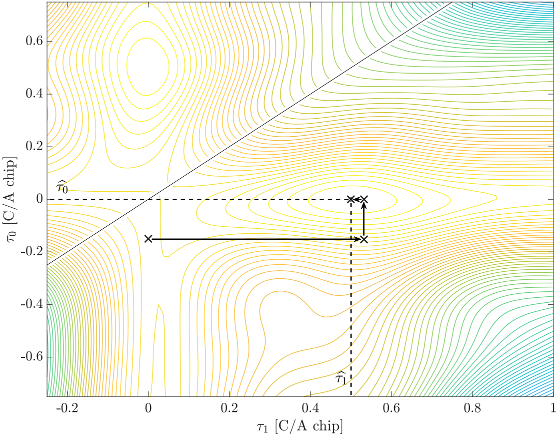

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Data Collection Campaign at Ayrolle Pond

Data Collection Campaign

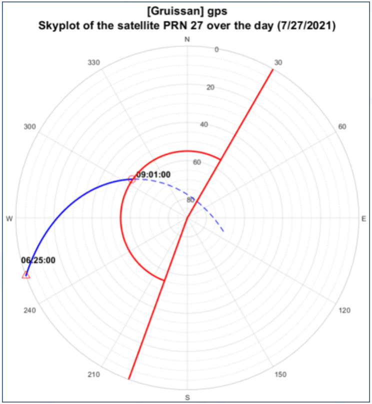

Figure: Predicted skyplot example.

- Site selection (CNES)
- Constellation state prediction with mask:
  - Two Line Elements (TLE),
  - SGP4 Orbit propagator,
  - Validation with existing online tools.*

*Thanks Dani from DLR!

Data Collection Campaign

Figure: Ayrolle Pond, near Gruissan on July 27, 2021.

- Site selection (CNES)
- Constellation state prediction with mask:
  - Two Line Elements (TLE),
  - SGP4 Orbit propagator,
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- Collection campaign,*

*Thanks Jean-Louis, FX and Laurent from CNES!

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- Site selection (CNES)
- Constellation state prediction with mask:
  - Two Line Elements (TLE),
  - SGP4 Orbit propagator,
  - Validation with existing online tools.
- Collection campaign,
- Real data processing...
  - Software to check the data (ISAE)*,
  - Apply the algorithms...

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*Thanks Benoit from ISAE/DEOS!

Outline

Conclusion

- GNSS-R is a research area with great potential:
  - New wideband GNSS signals allow a better performance in GNSS-R,
  - Coming space mission HydroGNSS to demonstrate the capabilities of a GNSS-R receiver to cover a wide range of applications (biomass, permafrost, sea state, etc).
- The mathematical framework has been derived in the case of specular reflection which allows to compare existing and new algorithms performance,
- Real data collected at Gruissan and expected to come and support numerical simulations.

Thank you for your attention!

References I

[1] *Understanding GPS/GNSS: Principle and Applications*, 3rd ed. Artech House, 2017.

[2] V. U. Zavorotny, S. Gleason, E. Cardellach, and A. Camps, "Tutorial on Remote Sensing Using GNSS Bistatic Radar of Opportunity," *IEEE Geoscience and Remote Sensing Magazine*, vol. 2, no. 4, pp. 8–45, 2014.

[3] S. Ribó, J. C. Arco-Fernández, E. Cardellach, F. Fabra, W. Li, O. Nogués-Correig, A. Rius, and M. Martín-Neira, "A Software-Defined GNSS Reflectometry Recording Receiver with Wide-Bandwidth, Multi-Band Capability and Digital Beam-Forming," *Remote Sensing*, vol. 9, no. 5, 2017.

References II

[4] M. A. Ribot, J.-C. Kucwaj, C. Botteron, S. Reboul, G. Stienne, J. Leclère, J.-B. Choquel, P.-A. Farine, and M. Benjelloun, "Normalized GNSS Interference Pattern Technique for Altimetry," *Sensors*, vol. 14, no. 6, pp. 10 234–10 257, 2014.

[5] S. F. Yau and Y. Bresler, "A Compact Cramér-Rao Bound Expression for Parametric Estimation of Superimposed Signals," *IEEE Transactions on Signal Processing*, vol. 40, no. 5, pp. 1226–1230, 5 1992.

[6] C. Lubeigt, L. Ortega, J. Vilà-Valls, L. Lestarquit, and E. Chaumette, "Joint Delay-Doppler Estimation Performance in a Dual Source Context," *Remote Sensing*, vol. 12, no. 23, p. 3894, 2020.

References III

[7] A. Renaux, P. Forster, E. Chaumette, and P. Larzabal, "On the High-SNR Conditional Maximum-Likelihood Estimator Full Statistical Characterization," *IEEE Trans. Signal Process.*, vol. 54, no. 12, pp. 4840 – 4843, 12 2006.

[8] R. D. Van Nee, "The Multipath Estimating Delay Lock Loop," in *IEEE Second International Symposium on Spread Spectrum Techniques and Applications*, 1992, pp. 39–42.

[9] I. Ziskind and M. Wax, "Maximum Likelihood Localization of Multiple Sources by Alternating Projection," *IEEE Transactions on Acoustics, Speech, and Signal Processing*, vol. 36, no. 10, pp. 1553–1560, 1988.

back-up: Dual Source Maximum Likelihood

$x \sim \mathcal{CN}(\mathbf{A}\alpha, \sigma_n^2 \mathbf{I}_N)$, therefore, the likelihood function is:

$$p(\mathbf{x}, \boldsymbol{\epsilon}) = \frac{1}{\left(\pi\sigma_n^2\right)^N} e^{-\frac{1}{\sigma_n^2}\|\mathbf{x}-\mathbf{A}\alpha\|^2}. \tag{9}$$

Maximizing (9) is equivalent to minimizing $\|\mathbf{x} - \mathbf{A}\alpha\|^2$. And with the projector $\mathbf{P_A} = \mathbf{A}\left(\mathbf{A}^H\mathbf{A}\right)^{-1}\mathbf{A}^H$,

$$\begin{aligned}\|\mathbf{x} - \mathbf{A}\alpha\|^2 &= \|\mathbf{P_A}(\mathbf{x} - \mathbf{A}\alpha)\|^2 + \left\|\mathbf{P_A^\perp}(\mathbf{x} - \mathbf{A}\alpha)\right\|^2 \\ &= \underbrace{\left\|\mathbf{A}\left(\left(\mathbf{A}^H\mathbf{A}\right)^{-1}\mathbf{A}^H\mathbf{x} - \alpha\right)\right\|^2}_{\text{null for } \alpha \text{ well chosen}} + \left\|\mathbf{P_A^\perp}\mathbf{x}\right\|^2.\end{aligned}$$

back-up: Dual Source Maximum Likelihood Estimator (cont'd)

So the 2S-MLE $\widehat{\boldsymbol{\epsilon}}$ is reduced to the search of the parameters $(\boldsymbol{\eta}_0, \boldsymbol{\eta}_1)$ that maximize the projection of the data upon the data subspace:

$$\widehat{\boldsymbol{\epsilon}} = \arg\max_{\boldsymbol{\epsilon}} p(\mathbf{x}, \boldsymbol{\epsilon})$$

$$\Leftrightarrow \widehat{\boldsymbol{\epsilon}} = \arg\min_{\boldsymbol{\epsilon}} \|\mathbf{x} - \mathbf{A}\alpha\|^2$$

$$\Leftrightarrow \begin{cases} \left(\widehat{\boldsymbol{\eta}_0}, \widehat{\boldsymbol{\eta}_1}\right) & = \arg\max_{\boldsymbol{\eta}_0, \boldsymbol{\eta}_1} \|\mathbf{P}_{\mathbf{A}}\mathbf{x}\|^2, \\ \widehat{\alpha} & = \left(\mathbf{A}^H\mathbf{A}\right)^{-1}\mathbf{A}^H\mathbf{x}, \\ \widehat{\sigma_n^2} & = \frac{1}{N}\left\|\mathbf{P}_{\mathbf{A}}^{\perp}\mathbf{x}\right\|^2. \end{cases}$$