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Close-to-Ground Single Antenna GNSS-R

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Outline

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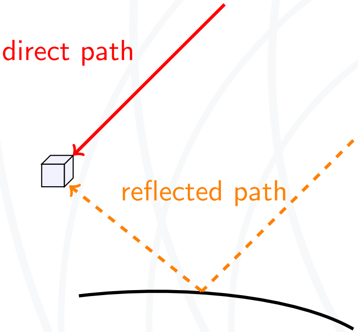
GNSS-Reflectometry

- GNSS signals (GPS, GALILEO and others) as signals of opportunity,
- Remote sensing: altimetry, biomass, wind speed, soil moisture, etc.

Spaceborne GNSS-R

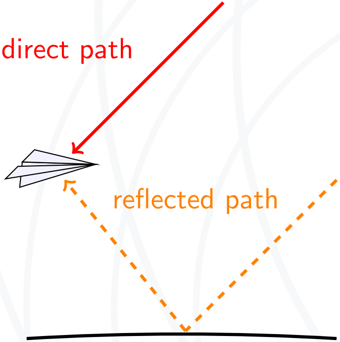


Spaceborne GNSS-R

- Low Earth Orbit satellites (TDS-1, CYGNSS, Hydro-GNSS),
- important coverage and revisit time*,
- mixture of coherent and non-coherent reflection (scattering),
- resolution due to the satellite motion.

*[1] Zavorotny et al "Tutorial on Remote Sensing Using GNSS Bistatic Radar of Opportunity," 2014.

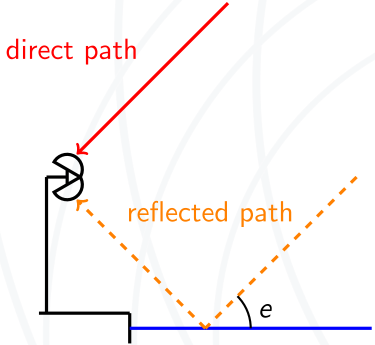
Airborne GNSS-R



Airborne GNSS-R

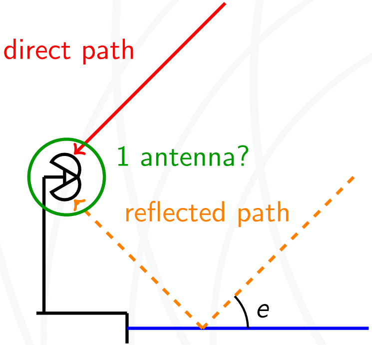
- various platforms: airplane, UAV, etc.
- better quality of the reflected signal,
- signal potentially more coherent,
- resolution due to the aircraft motion.

Ground-based GNSS-R



Ground-based GNSS-R

- snow cover, soil moisture and tide monitoring,
- static installation, local coverage,
- usually done with two antennas (upward and downward),



Ground-based GNSS-R

- snow cover, soil moisture and tide monitoring,
- static installation, local coverage,
- usually done with two antennas (upward and downward),
- GNSS Interferometric Reflectometry (GNSS-IR), coherent reflection.

Outline

Signal Model: Assumptions

- Dual source model with an assumed specular reflection:

$$\mathbf{x} = \mathbf{A}(\tau_0, \Delta\tau)\alpha + \mathbf{w},\ \mathbf{w} \sim \mathcal{CN}(0, \sigma_n^2 \mathbf{I}_N), \tag{1}$$

with

$$\mathbf{A}(\tau_0, \Delta\tau) = [\mathbf{s}(\tau_0),\, \mathbf{s}(\tau_0 + \Delta\tau)]\,,\ \boldsymbol{\alpha}^T = \left(\rho_0 e^{j\phi_0},\, \rho_1, e^{j\phi_1}\right).$$

- Doppler frequencies assumed equal and compensated due to the geometry (close-to-ground).
- Deterministic formulation with the following unknown vector:

$$\boldsymbol{\epsilon}^T = [\sigma_n^2, \underbrace{\tau_0, \rho_0, \phi_0}_{\boldsymbol{\theta}_0^T}, \underbrace{\Delta\tau, \rho_1, \phi_1}_{\boldsymbol{\theta}_1^T}] \tag{2}$$

Signal Model

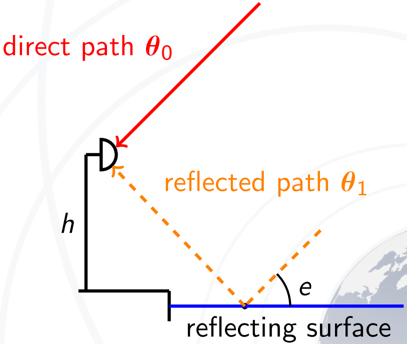


Figure: Geometry considered

Cramér-Rao Bounds (CRB)

From previous work*, the CRB for the estimation of $\boldsymbol{\epsilon}$ is obtained by inverting the corresponding Fisher Information Matrix:

$$\mathbf{F}_{\boldsymbol{\epsilon}|\boldsymbol{\epsilon}}(\boldsymbol{\epsilon}) = \begin{bmatrix} F_{\sigma_n^2|\boldsymbol{\epsilon}}(\boldsymbol{\epsilon}) & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{F}_{\theta_0|\boldsymbol{\epsilon}}(\boldsymbol{\epsilon}) & \mathbf{F}_{\theta_0,\theta_1|\boldsymbol{\epsilon}}(\boldsymbol{\epsilon}) \\ \mathbf{0} & \mathbf{F}_{\theta_1,\theta_0|\boldsymbol{\epsilon}}(\boldsymbol{\epsilon}) & \mathbf{F}_{\theta_1|\boldsymbol{\epsilon}}(\boldsymbol{\epsilon}) \end{bmatrix} \tag{3}$$

*[2] Lubeigt et al "Joint Delay-Doppler Estimation Performance in a Dual Source Context," 2020.

Outline

Prior Considerations: 2S-MLE

It can be shown that the Dual Source Maximum Likelihood Estimator (2S-MLE) associated to the present problem can be written as:

$$\left(\widehat{\tau_0}, \widehat{\Delta\tau}\right) = \arg\max_{\tau_0, \Delta\tau} \left\| \mathbf{A} \left(\mathbf{A}^H \mathbf{A} \right)^{-1} \mathbf{A}^H \mathbf{x} \right\|^2 = \arg\max_{\tau_0, \Delta\tau} L(\tau_0, \Delta\tau) \quad (4)$$

where $L(\tau_0, \Delta\tau)$ is the so-called likelihood criterion.

Reminder: Path Separation

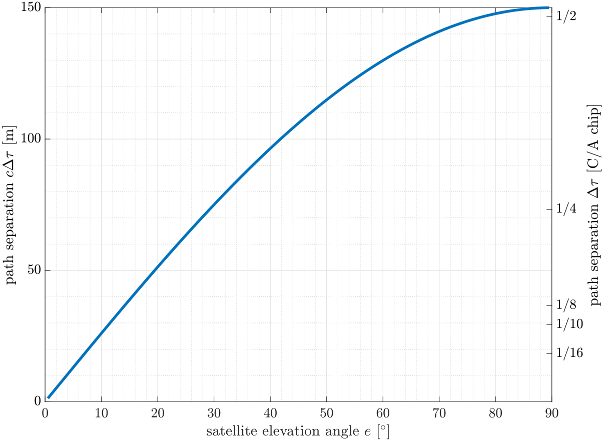


Figure: Path separation induced with regard to the satellite elevation angle for $h = 75$m.

Small Satellite Elevation Angle

- Assumption: e small $\Rightarrow \Delta\tau \ll 1$, then the auto-correlation function can be approximated by:

$$c(\Delta\tau) \approx \sum_k c_k \Delta\tau^k, \tag{5}$$

- From (5) and previous work*, the likelihood criterion can be approximated by a 2[nd] order Taylor polynomial:

$$L(\tau_0, \Delta\tau) \approx L^{\text{Taylor}}(\tau_0, \Delta\tau) = L_0(\tau_0) + L_1(\tau_0)\Delta\tau + L_2(\tau_0)\Delta\tau^2$$

*[3] Vincent et al "Approximate Maximum Likelihood Estimation of Two Closely Spaced Sources," 2014.

Approximate Maximum Likelihood Estimator

$$L(\tau_0, \Delta\tau) \approx L^{\text{Taylor}}(\tau_0, \Delta\tau) = L_0(\tau_0) + L_1(\tau_0)\Delta\tau + L_2(\tau_0)\Delta\tau^2$$

- Easily maximised w.r.t $\Delta\tau$ with $\Delta\tau(\tau_0) = -L_1(\tau_0)/2L_2(\tau_0)$ (zeroing the first derivative),
- The maximum likelihood reduces then to:

$$\widehat{\tau_0} = \arg\max_{\tau_0} L^{\text{Taylor}}(\tau_0, \Delta\tau(\tau_0)) \tag{6}$$

$$\text{and } \widehat{\Delta\tau} = -L_1(\widehat{\tau_0})/2L_2(\widehat{\tau_0}). \tag{7}$$

Reminder: Path Separation

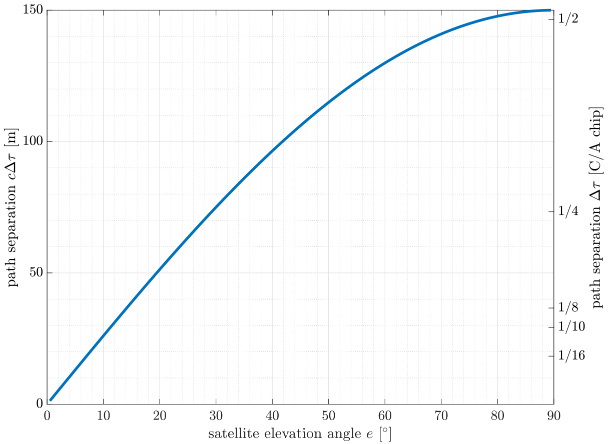


Figure: Path separation induced with regard to the satellite elevation angle for $h = 75$m.

Large Satellite Elevation Angle

- Possibility of separating the sources,
- existing algorithms: 2S-MLE, CLEAN-RELAX Estimator (MEDLL), or other from GNSS multipath mitigation techniques.
- usually, biased due to strong interference between the direct signal and the reflected signal.
- MPEE: Multipath Error Envelope to study the minimum path separation.

CLEAN-RELAX Estimator (CRE)

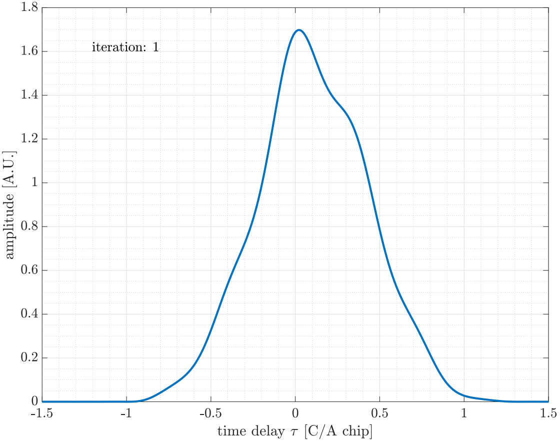


Figure: First estimation.

CLEAN-RELAX Estimator (CRE)

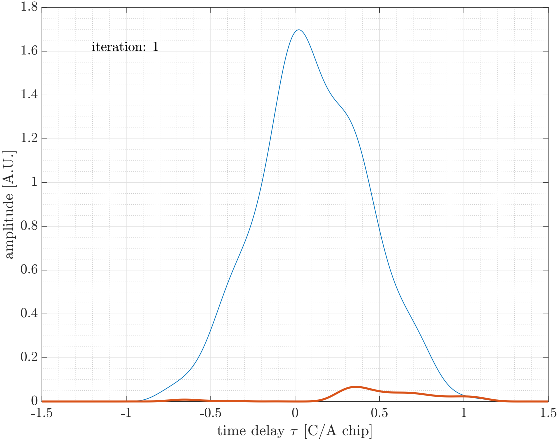


Figure: Second estimation upon the residue.

CLEAN-RELAX Estimator (CRE)

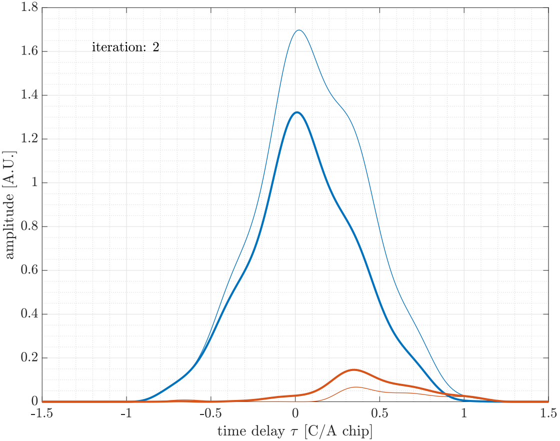


Figure: Iterate...

CLEAN-RELAX Estimator (CRE)

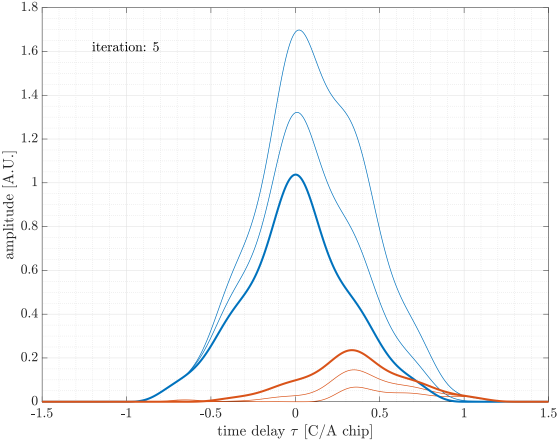


Figure: ... until convergence.

CLEAN-RELAX Estimator (CRE)

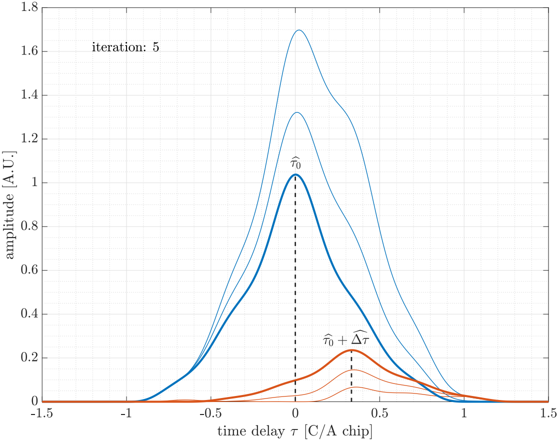


Figure: Read the estimate.

CLEAN-RELAX Estimator (CRE)

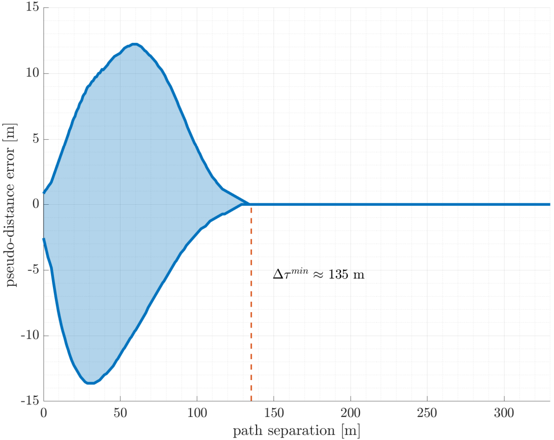


Figure: MPEE for the CRE with a GPS L1 C/A signal with an RF front-end bandwidth is set to 4 MHz and relative amplitude of 0.5.

Outline

Scenarios Considered

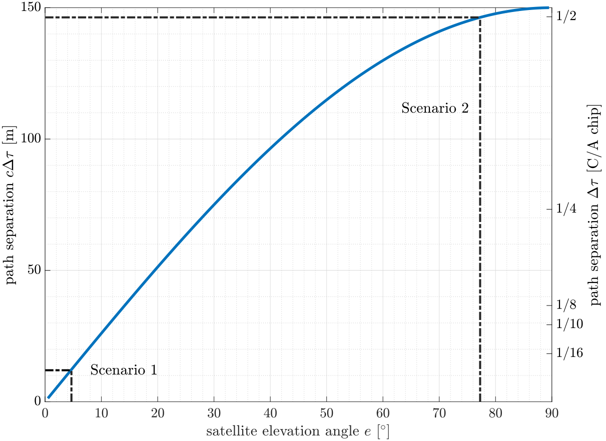


Figure: Scenario 1: $e_1 = 4.5°$, $c\Delta\tau = 11.7$m. Scenario 2: $e_2 = 77°$, $c\Delta\tau = 146.5$m.

Simulation Set-Up

- Signal: GPS L1 C/A,
- RF front-end bandwidth $B =$ sampling frequency $F_s = 4$ MHz,
- $\Delta\tau^1 = 11.2$m, $\Delta\tau^2 = 146.5$m,
- amplitude ratio $\rho_1/\rho_0 = 0.5$,
- relative phases considered: $\Delta\phi = \pi/3$ and $\Delta\phi = \pi/2$,
- Monte Carlo runs: $nMC = 2000$ to compute the RMSE of $\widehat{\Delta\tau}$,
- Definition of SNR_{out}:

$$\text{SNR}_{\text{out}} \triangleq \frac{\rho_0^2}{\sigma_n^2}\int_0^{T_I} |s(t)|^2 \mathrm{d}t = \left(\frac{C}{N_0}\right) T_I.$$

Scenario 1: Small Satellite Elevation Angle – $\Delta\tau = 11.7$m

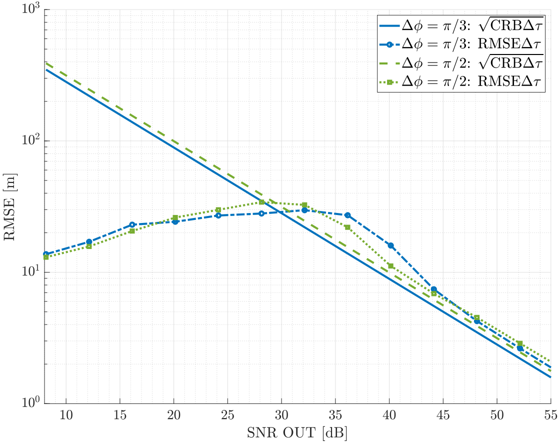


Figure: RMSE for estimation of $\Delta\tau$ with the AMLE.

Scenario 2: Large Satellite Elevation Angle – $\Delta\tau = 146.5$m

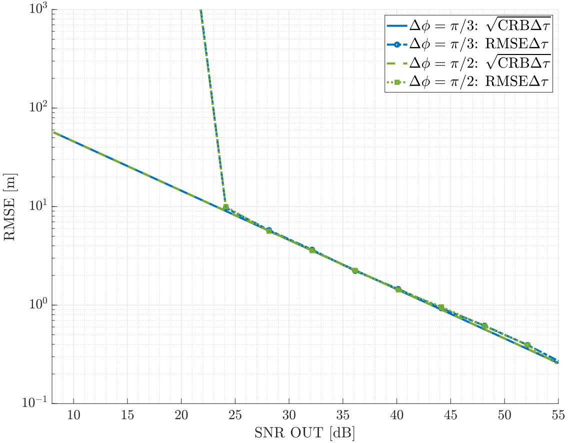


Figure: RMSE for estimation of $\Delta\tau$ with the CRE.

Outline

Conclusion

- Feasibility of close-to-ground GNSS-R altimetry.
- Small satellite elevation angle: Approximate Maximum Likelihood estimator:
  - 2^{nd} order Taylor expansion,
  - a small bias remains,
- Large satellite elevation angle: CLEAN-RELAX estimator:
  - presents good performance for narrowband GNSS signals,
  - wideband signals should allow to better separate sources and then widen the range of operation of such an estimator.

Et voilà

Thank you for your attention!

References I

[1] V. U. Zavorotny, S. Gleason, E. Cardellach, and A. Camps, "Tutorial on Remote Sensing Using GNSS Bistatic Radar of Opportunity," *IEEE Geoscience and Remote Sensing Magazine*, vol. 2, no. 4, pp. 8–45, 2014.

[2] C. Lubeigt, L. Ortega, J. Vilà-Valls, L. Lestarquit, and E. Chaumette, "Joint Delay-Doppler Estimation Performance in a Dual Source Context," *Remote Sensing*, vol. 12, no. 23, 2020. [Online]. Available: https://www.mdpi.com/2072-4292/12/23/3894

[3] F. Vincent, O. Besson, and E. Chaumette, "Approximate Maximum Likelihood Estimation of Two Closely Spaced Sources," *Signal Processing*, vol. 97, pp. 83–90, 2014. [Online]. Available: http://www.sciencedirect.com/science/article/pii/S0165168413004088

backup: Maximum Likelihood Estimator

$x \sim \mathcal{CN}(\mathbf{A}\alpha, \sigma_n^2 \mathbf{I}_N)$, therefore, the likelihood function is:

$$p(\mathbf{x}, \boldsymbol{\epsilon}) = \frac{1}{\left(\pi\sigma_n^2\right)^N} e^{-\frac{1}{\sigma_n^2}\|\mathbf{x}-\mathbf{A}\alpha\|^2}. \tag{8}$$

Maximising (8) is equivalent to minimising $\|\mathbf{x} - \mathbf{A}\alpha\|^2$. And with the projector $\mathbf{P_A} = \mathbf{A}\left(\mathbf{A}^H\mathbf{A}\right)^{-1}\mathbf{A}^H$,

$$\begin{aligned}\|\mathbf{x} - \mathbf{A}\alpha\|^2 &= \|\mathbf{P_A}(\mathbf{x} - \mathbf{A}\alpha)\|^2 + \left\|\mathbf{P}_\mathbf{A}^\perp(\mathbf{x} - \mathbf{A}\alpha)\right\|^2 \\ &= \underbrace{\left\|\mathbf{A}\left(\left(\mathbf{A}^H\mathbf{A}\right)^{-1}\mathbf{A}^H\mathbf{x} - \alpha\right)\right\|^2}_{\text{null for } \alpha \text{ well chosen}} + \left\|\mathbf{P}_\mathbf{A}^\perp\mathbf{x}\right\|^2.\end{aligned}$$

backup: Maximum Likelihood Estimator (cont'd)

$$\widehat{\boldsymbol{\epsilon}} = \arg\max_{\boldsymbol{\epsilon}} p(\mathbf{x}, \boldsymbol{\epsilon})$$

$$\Leftrightarrow \widehat{\boldsymbol{\epsilon}} = \arg\min_{\boldsymbol{\epsilon}} \|\mathbf{x} - \mathbf{A}\alpha\|^2$$

$$\Leftrightarrow \begin{cases} \left(\widehat{\tau_0}, \widehat{\Delta\tau}\right) & = \arg\max_{\tau_0, \Delta\tau} \|\mathbf{P}_{\mathbf{A}}\mathbf{x}\|^2 \\ \widehat{\alpha} & = \left(\mathbf{A}^H\mathbf{A}\right)^{-1} \mathbf{A}^H\mathbf{x} \\ \widehat{\sigma_n^2} & = \frac{1}{N} \left\|\mathbf{P}_{\mathbf{A}}^{\perp}\mathbf{x}\right\|^2 \end{cases}$$

backup: GNSS-R Altimetry

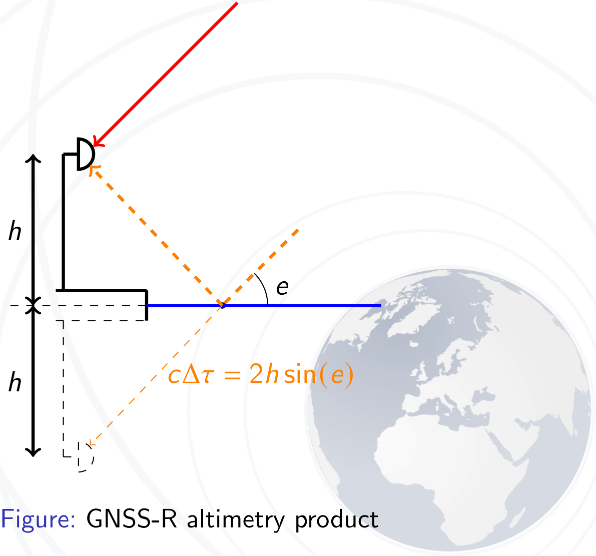


Figure: GNSS-R altimetry product

backup: Doppler Consideration

The phase difference between the direct and the reflected depends on the receiver's height and the satellite elevation:

$$\Delta\phi = \phi_1 - \phi_0 = \frac{2\omega_c h}{c} \sin(e). \tag{9}$$

Then, as the satellite elevation e varies, the relative phase varies and the first derivative corresponds to the relative Doppler frequency:

$$\Delta F_d = (b_1 - b_0) f_c = \frac{2 f_c h}{c} \cos(e) \frac{\mathrm{d}e}{\mathrm{d}t}. \tag{10}$$

For GPS L1 satellites with elevation rate $\mathrm{d}e/\mathrm{d}t = 0.14$ mrad/s, $e = 0$ rad, $f_c = 1.545$ GHz and a receiver at altitude $h = 75$ m: $\Delta F_d \approx 0.12$ Hz.