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Multipath Estimating Techniques Performance Analysis

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Outline

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Design of GNSS architectures

- Computational cost,
- Estimation accuracy,
- Robustness to harsh environment (multipath)
- etc.

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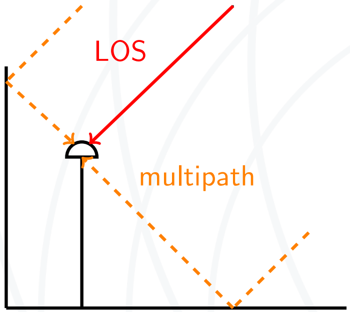
Multipath definition



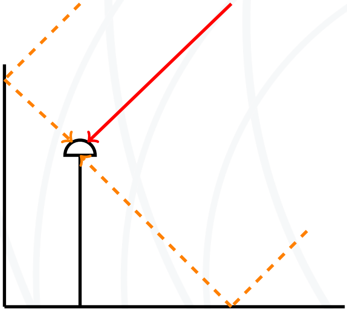
Definition*: *Multipath is the reception of multiple reflected or diffracted replicas of the desired signal, along with the direct path signal.*

- degradation of the estimation (bias induced),
- mobile application: random and dynamic phenomenon.

*[1] Kaplan and Hegarty, "Understanding GPS/GNSS: Principle and Applications," 2017.

Existing ways to deal with multipath

Hardware solutions



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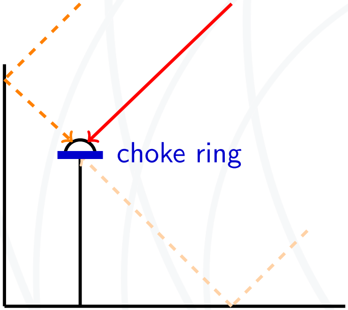

- choke ring

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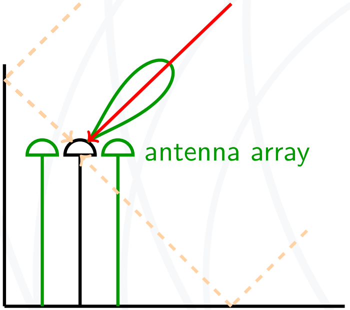

- choke ring
- array (beam-forming)

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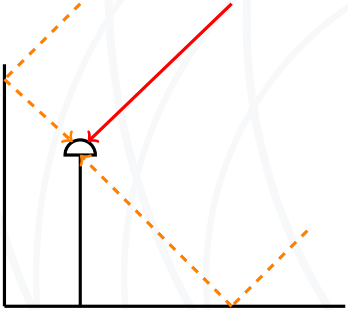

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Software solutions
(signal processing)

- based on the distortion of the ambiguity function,
- multipath estimating solutions.

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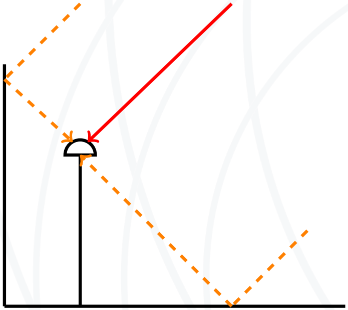

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Architecture Performance Analysis

- Multipath Error Envelope (MPEE)
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Architecture Performance Analysis

- Multipath Error Envelope (MPEE)
  - two-ray noise-free model
  - bias on the estimation of the direct signal's delay
  - does not allow to compare unbiased estimators performance with each other
- Unbiased estimators: Mean Square Error (MSE)
  - two-ray noisy model (SNR)
  - variance of the estimated direct signal's delay
  - universal Cramér-Rao lower bound

Outline

Signal Model: Assumptions

- Single multipath from a specular reflection:

$$x(t) = \rho_0 e^{j\phi_0} s(t - \tau_0) + \rho_1 e^{j\phi_1} s(t - \tau_1) + w(t) \qquad (1)$$

- White Gaussian noise: $w \sim \mathcal{N}(0, \sigma_n^2)$
- Deterministic formulation with the following unknown vector:

$$\boldsymbol{\epsilon}^T = [\sigma_n^2, \underbrace{\tau_0, \rho_0, \phi_0}_{\boldsymbol{\theta}_0^T}, \underbrace{\tau_1, \rho_1, \phi_1}_{\boldsymbol{\theta}_1^T}] \qquad (2)$$

Signal Model

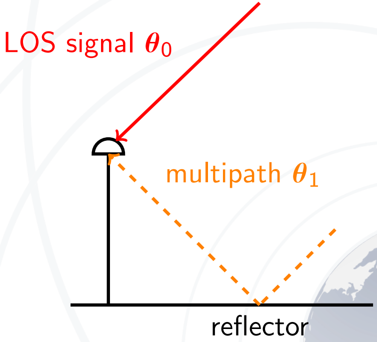

Figure: Geometry considered

Multipath Error Envelope (MPEE)

MPEE: range of the bias induced by a multipath upon the estimated $\widehat{\boldsymbol{\tau}}_0$. For a given estimator, it can be defined as follows:

$$\left\{\max_{\Delta\phi}\left(\text{bias}\left(\text{MDR}, \Delta\tau, \Delta\phi\right)\right), \min_{\Delta\phi}\left(\text{bias}\left(\text{MDR}, \Delta\tau, \Delta\phi\right)\right)\right\} \quad (3)$$

with

- $\text{MDR} = \rho_1/\rho_0$, the multipath-to-direct amplitudes ratio,
- $\Delta\tau = \tau_1 - \tau_0$, the multipath excess delay,
- $\Delta\phi = \phi_1 - \phi_0$, the relative phase.

MPEE: Example

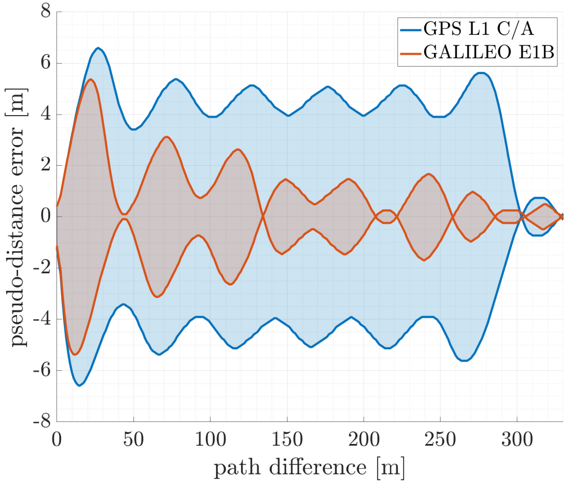

Figure: 1S-MLE MPEE for GPS L1 C/A and GALILEO E1B signals.

Cramér-Rao Bounds (CRB)

From previous work*, the CRB for the estimation of $\boldsymbol{\epsilon}$ is obtained by inverting the corresponding Fisher Information Matrix:

$$\mathbf{F}_{\boldsymbol{\epsilon}|\boldsymbol{\epsilon}}(\boldsymbol{\epsilon}) = \begin{bmatrix} F_{\sigma_n^2|\boldsymbol{\epsilon}}(\boldsymbol{\epsilon}) & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{F}_{\theta_0|\boldsymbol{\epsilon}}(\boldsymbol{\epsilon}) & \mathbf{F}_{\theta_0,\theta_1|\boldsymbol{\epsilon}}(\boldsymbol{\epsilon}) \\ \mathbf{0} & \mathbf{F}_{\theta_1,\theta_0|\boldsymbol{\epsilon}}(\boldsymbol{\epsilon}) & \mathbf{F}_{\theta_1|\boldsymbol{\epsilon}}(\boldsymbol{\epsilon}) \end{bmatrix} \quad (4)$$

*[2] Lubeigt et al "Joint Delay-Doppler Estimation Performance in a Dual Source Context," 2020.

Outline

Algorithms

- Maximum-likelihood-based methods:
  - **Multipath Estimating Delay Lock Loop (MEDLL or CLEAN-RELAX)***
  - Multipath Mitigation Technique (MMT or 2S-MLE)
  - Alternating Projection Estimator (APE)
- Correlator-based method:
  - Pulse Aperture Correlator (PAC)

*[3] Van Nee, "The Multipath Estimating Delay Lock Loop," 1992.
[4] Townsend et al, "Performance Evaluation of the Multipath Estimating Delay Lock Loop," 1995.

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*[5] Weill, "Multipath Mitigation Using Modernized GPS Signal: How Good Can It Get?" 2002.

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*[6] Ziskind and Wax, "Maximum Likelihood Localization of Multiple Sources by Alternating Projection," 1988.

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*[7] Jones et al, "Theory and Performance of the Pulse Aperture Correlator," 2004.

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MPEE

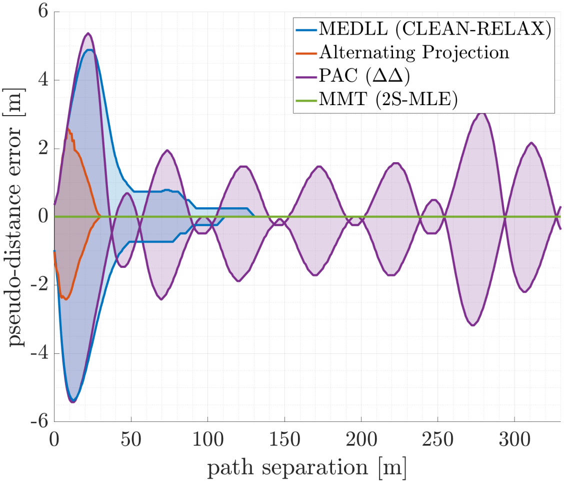

Figure: MPEE for GPS L1 C/A.

RMSE w.r.t. the path separation

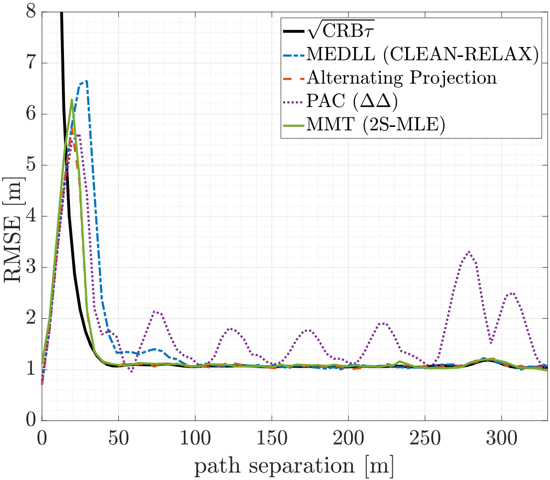

Figure: RMSE of $\widehat{\tau_0}$ w.r.t. $c\Delta\tau$ for GPS L1 C/A and $\mathrm{SNR}_{\mathrm{out}} = 31\mathrm{dB}$.

RMSE w.r.t. the SNR

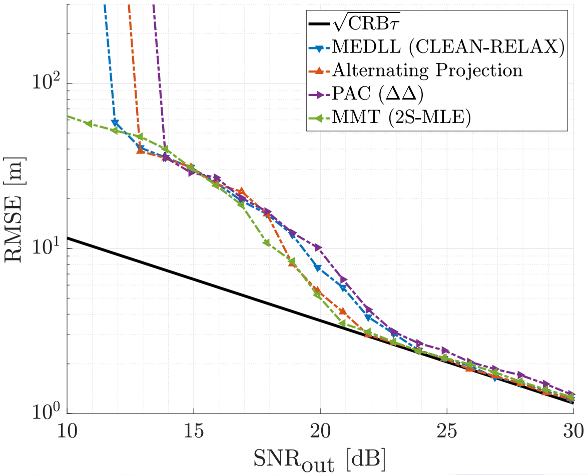

Figure: RMSE of $\widehat{\tau_0}$ w.r.t. $\mathrm{SNR_{out}}$ for GPS L1 C/A and $c\Delta\tau = 150$m.

Outline

Conclusion

- MPEE approach:
  - graphical tool, easy to read,
  - information on the bias only,
  - assumed a noise-free environment.
- RMSE approach:
  - information on the variance,
  - can be compared to a theoretical lower bound,
  - takes into account the noise,
  - provides operating point in term of minimum SNR and path separation,
  - not a general solution (since a SNR or a path separation needs to be set).
- Through these two combined approaches, the Alternating Projection Estimator seems an excellent candidate as a multipath mitigation technique.

Et voilà

Thank you for your attention!

References I

[1] E. Kaplan and C. Hegarty, *Understanding GPS/GNSS: Principle and Applications*, 3rd ed. Artech House, 2017.

[2] C. Lubeigt, L. Ortega, J. Vilà-Valls, L. Lestarquit, and E. Chaumette, "Joint Delay-Doppler Estimation Performance in a Dual Source Context," *Remote Sensing*, vol. 12, no. 23, 2020. [Online]. Available: https://www.mdpi.com/2072-4292/12/23/3894

[3] R. D. Van Nee, "The Multipath Estimating Delay Lock Loop," in *IEEE Second International Symposium on Spread Spectrum Techniques and Applications*, 1992, pp. 39–42.

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[4] B. R. Townsend, P. C. Fenton, K. J. Van Dierendonck, and R. D. J. Van Nee, "Performance Evaluation of the Multipath Estimating Delay Lock Loop," *Navigation*, vol. 42, no. 3, pp. 502–514, 1995. [Online]. Available: https://onlinelibrary.wiley.com/doi/abs/10.1002/j.2161-4296.1995.tb01903.x

[5] L. R. Weill, "Multipath Mitigation using Modernized GPS Signals: How Good Can it Get?" *Proceedings of the 15th International Technical Meeting of the Satellite Division of The Institute of Navigation (ION GPS 2002)*, pp. 493–505, September 2002.

[6] I. Ziskind and M. Wax, "Maximum Likelihood Localization of Multiple Sources by Alternating Projection," *IEEE Transactions on Acoustics, Speech, and Signal Processing*, vol. 36, no. 10, pp. 1553–1560, 1988.

References III

[7] J. Jones, P. Fenton, and B. Smith, “Theory and Performance of the Pulse Aperture Correlator,” NovAtel Inc., Tech. Rep., 2004. [Online]. Available: https://hexagondownloads.blob.core.windows.net/public/Novatel/assets/Documents/Papers/PAC/PAC.pdf

backup: Simulation set-up

- Signals: GPS L1 C/A and GALILEO E1B,
- pre-correlation bandwidth $B =$ sampling frequency $F_s = 12 \times 1.023$ MHz,
- Multipath-to-direct amplitudes ratio MDR $= 0.5$,
- for RMSE w.r.t. SNR, relative phase $\Delta\phi = 0$,
- Monte Carlo runs: $nMC = 1000$
- Definition of SNR_{out}:

$$\text{SNR}_{\text{out}} \triangleq \frac{\rho_0^2}{\sigma_n^2} \int_0^{T_I} |s(t)|^2 \mathrm{d}t = \left(\frac{C}{N_0}\right) T_I \tag{5}$$

backup: MPEE

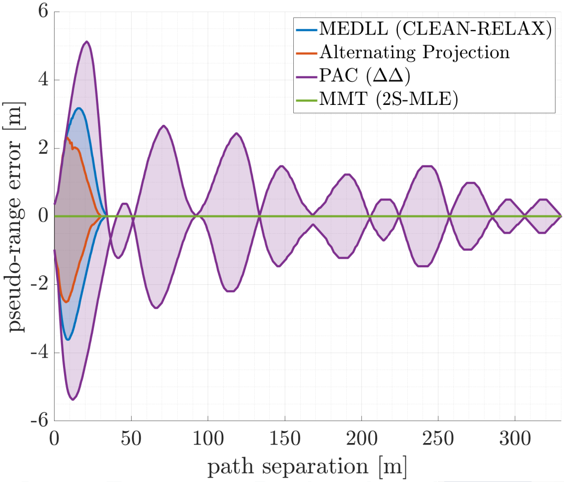

Figure: MPEE for GALILEO E1B.

backup: RMSE w.r.t. the path separation

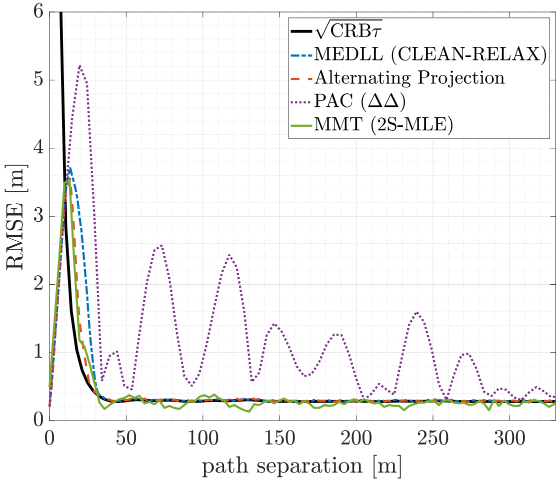

Figure: RMSE of $\widehat{\tau}_0$ w.r.t. $c\Delta\tau$ for GALIELO E1B and $\mathrm{SNR}_{\mathrm{out}} = 34\mathrm{dB}$.

backup: RMSE w.r.t. the SNR

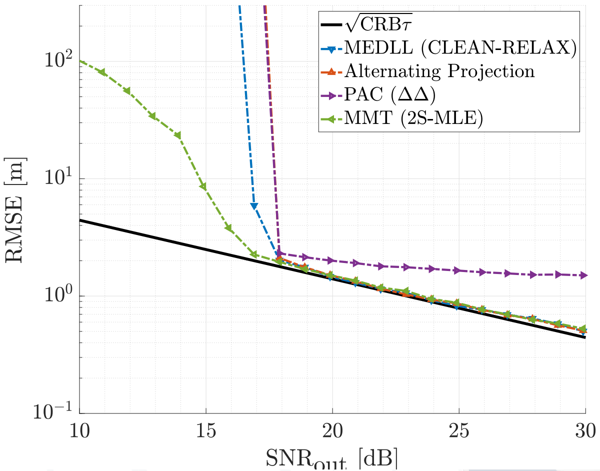

Figure: RMSE of $\widehat{\tau}_0$ w.r.t. SNR_{out} for GALILEO E1B and $c\Delta\tau = 150\text{m}$.